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Three-dimensional non-grey gas radiative transfer analyses using the statistical narrow-band model

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Abstract—Three-dimensional non-grey gas radiation analyses were conducted using the statistical narrow-band model with updated band parameters and three implementation methods: the exact or the correlated formulation, the non-correlated expression, and the grey-band approximation with the absorption coefficient estimated using the local properties. The accuracy of the two approximate narrow-band implementation methods was evaluated for both low-resolution spectral intensity and spectrally integrated radiative source term and wall heat flux in a rectangular enclosure containing (1) isothermal pure water and (2) a CO₂/H₂O/N₂ mixture with a furnace type gas temperature distribution. For spectrally integrated quantities, results of the grey-band approximation are very close to those of the non-correlated formulation and are in qualitative agreement with the results of the correlated formulation. The two approximate methods are capable of predicting qualitatively correct and fairly accurate distributions of low-resolution spectral radiation intensities for the isothermal case. However, they predict less accurate low-resolution spectral intensities for the non-isothermal case. © Elsevier, Paris

non-grey gas radiation / statistical narrow-band model / low-resolution spectral radiation intensity / ray-tracing / discrete-ordinates method / correlated and non-correlated formulation / grey-band approximation

Résumé — Analyses du transfert radiatif tri-dimensionnel dans un gaz non gris utilisant le modèle statistique de bande étroite. Des analyses du rayonnement tri-dimensionnel dans un gaz non gris ont été effectuées en utilisant le modèle statistique de bande étroite, avec des paramètres de bande actualisés et trois méthodes de mise en œuvre : la formulation exacte ou corrélée, l'expression non corrélée et l'approximation de bande grise, le coefficient d'absorption étant estimé en faisant appel aux propriétés locales. La précision des deux méthodes utilisant l'approximation de bande étroite a été évaluée à la fois pour l'intensité spectrale basse résolution, le terme de source radiative spectralement intégré et le flux de chaleur aux parois dans une enceinte rectangulaire contenant (1) de l'eau pure à température constante et (2) un mélange CO₂/H₂O/N₂, avec une distribution de température dans le gaz de type four. En ce qui concerne les quantités intégrées spectralement, les résultats de l'approximation de bande grise sont très proches de ceux obtenus par la formulation non corrélée et sont en accord qualitatif avec ceux obtenus par la formulation corrélée. Les deux méthodes d'approximation sont capables de prévoir des distributions qualitativement correctes et assez précises des intensités de radiation spectrale basse résolution pour le cas isotherme. Cependant, elle prévoit les intensités de radiation spectrale basse résolution avec une précision moindre pour le cas isotherme. © Elsevier, Paris

radiation dans un gaz non gris / modèle statistique de bande étroite / intensité spectrale de radiation basse résolution / traçage radiatif / méthode des ordonnées discrètes / formulations corrélée et non corrélée / approximation de bande grise

Nomenclature

E	average spectral error
f	species molar fraction
I_ν	spectral radiation intensity W·m ⁻² ·cm·sr ⁻¹
$\bar{k}_\nu$	mean line-intensity to spacing ratio cm ⁻¹ ·atm ⁻¹
l_m	local mean pathlength m

$\hat{n}$	unit normal vector of a wall surface	
p	pressure.....	atm
q	heat flux density vector	W·m ⁻²
S	surface area of a computational grid.....	m ²
S_r	source term due to radiative transfer	W·m ⁻³
s, s'	position variables	m
T	temperature.....	K
V	volume of a computational grid.....	m ³
w_n	weight function associated with the n th direction	

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x, y, z cartesian coordinates..... m

Greek symbols

$\bar{\beta}_\nu$ mean line-width to spacing ratio
 ε_w wall surface emissivity
 $\bar{\kappa}_\nu$ narrow-band averaged absorption coefficient..... m^{-1}
 $\bar{\gamma}_\nu$ mean half-width of an absorption line..... cm^{-1}
 $\bar{\delta}_\nu$ equivalent line spacing..... cm^{-1}
 ν wavenumber..... cm^{-1}
 $\Delta\nu$ wavenumber interval of a narrow-band..... cm^{-1}
 τ_ν spectral transmissivity
 ξ, η, μ direction cosines
 Ω direction or solid angle..... sr

Subscripts

b blackbody
i spatial discretisation (along a line of sight) index
n angular discretisation index
w wall
 ν spectral

1. INTRODUCTION

Accurate calculation of total (spectrally integrated) quantities of radiative transfer (wall heat flux and radiative source term) is important for modelling large-scale combustion and flame phenomena, yet it is a difficult task primarily due to the extremely strong spectral dependency of radiative properties of radiating gases. On the other hand, computational efficiency is also an important concern when dealing with practical problems. As far as the calculation of total radiative quantities is concerned, the spectral line-based weighted-sum-of-grey-gases model (SLWSGG) developed by Denison and Webb [1–3] is perhaps the most favoured model for handling non-grey radiative transfer over the correlated- k method and various band models in terms of accuracy, ease of implementation, and computing efficiency. Recently, Kim and Song [4] have extended the application of the conventional WSGG model [5] to a narrow-band. Although the narrow-band WSGG model is suitable for some specific applications where low-resolution spectral intensity is to be obtained, such as the prediction of low-resolution infrared (IR) signature from a rocket plume, its computational efficiency is certainly not comparable to the SLWSGG model in the calculation of total radiative quantities. Of course, the current SLWSGG model can also be extended to a narrow-band for the calculation of low-resolution spectral intensity.

Band models (wide-band and narrow-band) are in general capable of providing accurate band averaged

gas transmissivities. They also represent a potentially promising compromise between accuracy and computational efficiency for engineering applications in view of the rapid advance in computer speed. However, they have not been widely used mainly due to the following two reasons. First, band models provide a band averaged transmissivity rather than the fundamental property, i.e. the gas absorption coefficient. Consequently, most of the solution methods cannot be readily used with the exceptions of Monte Carlo methods and the discrete transfer method. Secondly, the rigorous implementation of a band model into the radiative transfer equation (RTE) leads to a formidable correlation term between the spectral gas absorption coefficient and the spectral radiation intensity which significantly complicates the band averaged RTE and requires much more computing time. In addition, neglect of the correlation term could cause considerable errors. The correlated- k ($c-k$) method represents an alternative between the line-by-line (LBL) and narrow-band models [6]. Like the narrow-band WSGG model, direct application of this method requires a quadrature inside each narrow-band. Hence, the $c-k$ method and the narrow-band WSGG method could be even more computationally expensive than a narrow-band model, albeit they are compatible with the standard RTE.

Occasionally, the objectives of a study are not only to obtain the total radiative quantities, but also the low-resolution (on the order of a narrow-band) distribution of spectral radiation intensity along a line-of-sight. An obvious example of this type of application is the prediction of IR signature from a rocket plume, as mentioned earlier. In this situation, the current SLWSGG and wide-band models cannot be used, since they are not able to offer the required spectral resolution, and the employment of a narrow-band model, the $c-k$ method, or the narrow-band WSGG model becomes necessary.

In order to overcome the difficulties associated with the rigorous implementation of a narrow-band model into the RTE [7, 8], two approximate implementation methods have been developed. One is the non-correlated formulation proposed by De Miranda and Sacadura [9]. The other is the various versions of the grey-band model [7, 8, 10, 11]. At a spectral interval of a narrow-band, the grey-band approximation represents the simplest and most efficient model compared to the $c-k$ method or the narrow-band WSGG model, since no further details of gas radiative properties inside each narrow-band need to be resolved. This is analogous to the grey gas approximation made in the calculation of total radiative quantities. Although the non-correlated formulation of a narrow-band model suggested by De Miranda and Sacadura [9] significantly improves the computational efficiency compared to the correlated formulation, its solution in three-dimensions is difficult, as absorption coefficient based methods are not applicable. In contrast, the grey-band model can be solved by using an arbitrary solution technique and is

therefore favoured over the non-correlated formulation in three-dimensions.

To implement a narrow-band model via the grey-band model, a path-length must be specified a priori in order to extract an absorption coefficient from the transmissivity, using the Beer-Lambert relation. However, an obvious choice of path-length may not exist in complex multidimensional media [3]. This constitutes another objection to the application of a band model. Nevertheless, some efforts have been made in the literature to extract a gas absorption coefficient from a band transmissivity. Fiveland and Jamaluddin [10] formulated a grey wide-band model where the wide-band gas absorption coefficient was extracted from the Beer-Lambert relation, using the wide-band gas transmissivity and the global mean path-length of the geometry. The model was incorporated into the discrete-ordinates method (DOM) to solve radiative transfer in three-dimensional absorbing, emitting, and isotropically scattering gas-particle mixtures. Another grey wide-band model was formulated by Modest and Sikka [11] for radiative transfer in multidimensional absorbing, emitting, and linear-anisotropically scattering media. Modest and Sikka [11], unlike Fiveland and Jamaluddin [10], obtained the equivalent bandwidth and the equivalent gas absorption coefficient from the band absorptances at the optically transparent limit and at the mean path-length of the geometry of the medium under consideration. This model was then implemented into the $P-1$ approximation, with some calculations in one-dimension performed. Kim et al. [7] also conducted grey narrow-band and wide-band calculations with the gas absorption coefficient extracted from the transmissivity based on the global mean path-length. Their results showed that the use of the global mean path-length even fails to predict the correct trend of radiative source term distribution in one-dimension parallel-plate geometry. In general, the specification of a proper mean path-length for the extraction of gas absorption coefficient in three-dimensional complex geometry is difficult. More recently, Liu et al. [8] developed a grey narrow-band model in which the gas absorption coefficient is calculated based on the mean path-length of the local computational grid. They showed for several one-dimensional problems that this grey-band approximation yields total quantities in qualitative agreement with those of the correlated method. In view of its high computational efficiency and compatibility with the standard RTE, the grey-band approximation is favoured as a feasible approach to implement a narrow-band model, even with some loss of accuracy, for problems where low-resolution spectral radiation intensities are to be obtained.

The objectives of the present study are to further evaluate the accuracy of the two approximate implementation methods of a narrow-band model in three-dimensions for both spectrally integrated quantities and low-resolution spectral radiation intensity. The benchmark solution used in the evaluation was the results

obtained by solving the correlated formulation. The narrow-band model employed in this study is the statistical narrow-band (SNB) model which has been widely used due to its accuracy and availability of band parameters. Radiation scattering due to particles is not considered in this work and will be the objective of a future study. Numerical calculations were conducted in a rectangular enclosure for two cases. In the first case, the medium is pure water vapour at 1 atm and 1 000 K. In the second case, the medium is a $\text{CO}_2/\text{H}_2\text{O}/\text{N}_2$ mixture at 1 atm and a furnace type gas temperature field is specified. Finally, low-resolution spectral intensities were also calculated in several one-dimensional problems in order to provide a direct comparison between the average spectral errors of the present results and those reported by Kim and Song [4] using narrow-band WSGG methods.

2. FORMULATION

Formulation of the three methods for implementing the SNB model is briefly summarised below, since detailed description of each method has been given in references [7–9]. In essence, the correlated formulation is the exact implementation method that treats the correlation term between the spectral absorption coefficient and spectral radiation intensity rigorously. The only assumption required is the high wall emissivity [7]. The non-correlated formulation was derived from the correlated one by assuming that the band transmissivity obeys Beer's law [9]. The grey-band model evaluated in this work was developed from the non-correlated formulation by making use of the mean path-length of the local computational grid [8].

2.1. Correlated narrow-band formulation

The discretized form of the rigorous narrow-band averaged RTE along a line-of-sight for high-emissivity surrounding walls is given as [7]

$$\bar{I}_{\nu,n,i+1} = \bar{I}_{\nu,n,i} + (1 - \bar{\tau}_{\nu,n,i \rightarrow i+1}) \bar{I}_{b\nu,i+1/2} + \bar{C}_{\nu,n,i+1/2} \quad (1)$$

where the overbar denotes quantities averaged over a narrow-band and

$$\begin{aligned} \bar{C}_{\nu,n,i+1/2} = & \bar{I}_{w\nu,n,1} (\bar{\tau}_{\nu,n,1 \rightarrow i+1} - \bar{\tau}_{\nu,n,1 \rightarrow i}) \\ & + \sum_{k=1}^{i-1} [(\bar{\tau}_{\nu,n,k+1 \rightarrow i+1} - \bar{\tau}_{\nu,n,k+1 \rightarrow i}) \\ & - (\bar{\tau}_{\nu,n,k \rightarrow i+1} - \bar{\tau}_{\nu,n,k \rightarrow i})] \bar{I}_{b\nu,k+1/2} \quad (2) \end{aligned}$$

which arises from the correlation of the spectral gas absorption coefficient and the spectral radiation intensity. This term dramatically increases the computing time of the correlated formulation [7, 8], since the narrow-band averaged radiation intensity at s_{i+1} depends not only on

the local properties ($\bar{I}_{\nu,n,i}$, $\bar{\tau}_{\nu,n,i \rightarrow i+1}$, and $\bar{I}_{b\nu,i+1/2}$), but also on properties between s_1 and s_i . The origin of this complicated source term is that the narrow-band averaged gas transmissivity does not obey the Beer-Lambert relation (exponential attenuation with the pathlength) [6]. The spatial discretisation index $i = 1$ corresponds to the starting point of the line-of-sight on a wall boundary.

The boundary spectral radiation intensity $\bar{I}_{\nu}(s_w, \Omega)$ at a diffuse wall is

$$\bar{I}_{\nu}(s_w, \Omega) = \varepsilon_{w\nu} \bar{I}_{bw\nu} + \frac{(1 - \varepsilon_{w\nu})}{\pi} \int_{2\pi} |\hat{n} \cdot \Omega'| \bar{I}_{\nu}(s_w, \Omega') d\Omega', \text{ for } \hat{n} \cdot \Omega' > 0 \quad (3)$$

2.2. Non-correlated narrow-band formulation

Starting from the correlated formulation, equation (1), and making an approximation that the band averaged gas transmissivity obeys the Beer-Lambert relation, De Miranda and Sacadura [9] obtained the recursive form of the non-correlated narrow-band averaged RTE:

$$\bar{I}_{\nu,n,i+1} = \bar{I}_{\nu,n,i} \bar{\tau}_{\nu,n,i \rightarrow i+1} + \bar{I}_{b\nu,i+1/2} (1 - \bar{\tau}_{\nu,n,i \rightarrow i+1}) \quad (4)$$

In this non-correlated formulation, the radiation intensity at s_{i+1} is dependent only on its immediate upstream properties. Therefore, this formulation offers much higher computational efficiency as the time-consuming source term, $\bar{C}_{\nu,n,i+1/2}$, disappears as a result of assuming the Beer-Lambert relation. Note that this is also the origin of error of the non-correlated formulation.

In the correlated and non-correlated formulations, the total (spectrally integrated) net radiative flux is calculated at the centre of each control surface using the following expression once the spectral radiation intensity field is obtained:

$$q_i = \sum_{\text{all } \Delta\nu} \left(\sum_{n=1}^N l_i \bar{I}_{\nu,n,i} w_n \right) \Delta\nu \quad (5)$$

where l_i is the direction cosine along i th direction, and $i = 1, 2$, and 3 correspond respectively to the x , y , and z directions. The radiative source term S_r is then obtained from

$$S_r = -\nabla \cdot \mathbf{q} \quad (6)$$

2.3. Grey-band formulation

For grey-band calculations, the following form of the RTE is solved [8]

$$\frac{\partial \bar{I}_{\nu}(s, \Omega)}{\partial s} = -\bar{\kappa}_{\nu}(s) \bar{I}_{\nu}(s, \Omega) + \bar{\kappa}_{\nu}(s) \bar{I}_{b\nu}(s) \quad (7)$$

Although this equation is the same as that solved by previous researchers [7, 10, 11], the present grey-band model differs from the previous one in how the mean gas absorption coefficient is calculated. Based on our previous work [8], an equivalent path independent narrow-band averaged local absorption coefficient is estimated using the mean path-length of the control volume under consideration

$$\bar{\kappa}_{\nu} = -\frac{\ln \bar{\tau}_{\nu}(l_m)}{l_m} \quad (8)$$

where $l_m = 3.6 V/S$ is the mean path-length of the local computational control volume [5].

The boundary condition is the same as that given in equation (3). The radiative source term is calculated from

$$S_r = \sum_{\text{all } \Delta\nu} \bar{\kappa}_{\nu} \left(4\pi \bar{I}_{b\nu} - \int_{4\pi} \bar{I}_{\nu} d\Omega \right) \Delta\nu \quad (9)$$

2.4. The statistical narrow-band model

The statistical narrow-band (SNB) model was used in the present study to calculate the narrow-band averaged gas transmissivities. In this model, the narrow-band averaged transmissivity for an isothermal and homogeneous path $|s' \rightarrow s|$ is given as [12]

$$\bar{\tau}_{\nu}(s' \rightarrow s) = \exp \left[-\frac{\bar{\beta}_{\nu}}{\pi} \left(\sqrt{1 + \frac{2\pi f p |s' \rightarrow s| \bar{k}_{\nu}}{\bar{\beta}_{\nu}}} - 1 \right) \right] \quad (10)$$

where the average line-width to spacing ratio is given as $\bar{\beta}_{\nu} = 2\pi \bar{\gamma}_{\nu} / \bar{\delta}_{\nu}$. For a non-isothermal and/or inhomogeneous path, the Curtis-Godson approximation [13] is used. It is worth noting that this approximation is required only in the calculations of the correlated formulation. The updated SNB model parameters provided by Soufiani [14] were used in the calculations of this study. This data set contains SNB model parameters for CO, CO₂, and H₂O with a constant spectral interval of 25 cm⁻¹ wide. The covered temperature and spectral ranges are 300 to 2 900 K and 150 to 9 300 cm⁻¹, respectively. Further details of the updated data set were described in reference [15].

3. RESULTS AND DISCUSSION

The correlated and non-correlated narrow-band averaged RTEs, equations (1) and (4), were solved using a ray-tracing method described in reference [16].

This method is essentially a modified version of the discrete transfer method to improve its accuracy in the calculation of the radiative source term at the cost of more computing time. The grey-band RTE, equation (7), was solved using DOM along with the positive (variable weighted) spatial differencing scheme [17] to avoid non-physical oscillations in the radiation intensity of the diamond scheme and offer better numerical accuracy than the step scheme.

Numerical calculations were performed in a $2 \times 2 \times 4$ m rectangular enclosure for two cases. In the first case, the medium in the enclosure is pure water vapour at 1 atm and at a uniform temperature of 1 000 K. The second case is more relevant to radiative transfer in a gas-fired furnace. In this case, the medium is assumed to be a uniform mixture of 0.1 CO₂ + 0.2 H₂O + 0.7 N₂ (mole basis) at 1 atm. The gas temperature is non-uniform but symmetrical about the centerline of the enclosure and is specified in terms of $T = (T_c - T_e)f(r/R) + T_e$. In this equation, T_c is the gas temperature along the centerline of the enclosure, T_e is the exit temperature at $z = 4$ m. Inside the circular region of the cross section of the enclosure, the variation of gas temperature is defined by $f(r/R) = 1 - 3(r/R)^2 + 2(r/R)^3$, where r is the distance from the enclosure centerline and R is the radius of the circular region ($R = 1$ m). The gas temperature outside the circular region is assumed to be uniform and at the value of the exit temperature. The centerline temperature is assumed to increase linearly from 400 K at the inlet ($z = 0$) to 1 800 K at $z = 0.375$ m, then decreases linearly to 800 K at the exit.

All wall surfaces are assumed to be black and cold (0 K in the first case and 300 K in the second case). All the calculations were carried out on a SGI workstation (Power Challenge, Model L) using a $11 \times 11 \times 16$ grid and T_4 (128 directions) quadrature set [18]. Uniform grids were used in the first case and non-uniform grids in z -direction were employed in the second case with finer grids placed around the peak gas temperature ($z = 0.375$ m). It has been checked that further spatial or angular grid refinement does not significantly alter the numerical results.

3.1. Spectrally integrated quantities

The calculated radiative source term distributions along the centerline of the enclosure, $(1 \text{ m}, 1 \text{ m}, z)$, and along $(x, 1 \text{ m}, 0.375 \text{ m})$ for the isothermal case are shown in figures 1 and 2, respectively. The two approximate methods underpredict the radiative source term by as much as 33 % (by the grey-band model) (figure 2). Results of the two approximate methods are quite close to each other. In addition, the non-correlated formulation in general predicts slightly more accurate radiative source terms than the grey-band model.

Distributions of wall heat flux density along $(2 \text{ m}, 1 \text{ m}, z)$ and $(x, 1 \text{ m}, 4 \text{ m})$ for the isothermal case are shown in figures 3 and 4. The two approximate methods

Figure 1. Distributions of the predicted radiative source term along $(1 \text{ m}, 1 \text{ m}, z)$ for the isothermal case.

Figure 2. Distributions of the predicted radiative source term along $(x, 1 \text{ m}, 0.375 \text{ m})$ for the isothermal case.

predict wall heat flux densities in very close agreement with each other and higher than the correlated results by as much as 11 %.

Results shown in figures 1–4 confirm that the non-correlated formulation and the present grey-band model also predict very similar spectrally integrated quantities in three-dimensions. This is actually expected from the development of the present grey-band approximation outlined in reference [8].

Figure 5 shows the distributions of the radiative source term along the centreline of the enclosure for the non-isothermal case. The two approximate methods reproduce the correct trend of the sharp variation of the source term along the centreline. Results of the two approximate methods are almost identical and

Figure 3. Distributions of the predicted wall heat flux density along (2 m, 1 m, z) for the isothermal case.

Figure 5. Distributions of the predicted radiative source term along (1 m, 1 m, z) for the non-isothermal case.

Figure 4. Distributions of the predicted wall heat flux density along (x , 1 m, 4 m) for the isothermal case.

in fairly good agreement with the correlated results. At the peak gas temperature, $z = 0.375$ m, the error of the approximate methods is about 13 %. The positive values found near the inlet ($z = 0$) imply that the gas actually receives radiant energy due to its low temperatures. Distributions of the radiative source term along (x , 1 m, 0.15 m), near the inlet, and (x , 1 m, 3.55 m), near the exit, are shown in figure 6. Similar to the isothermal case (figures 1 and 2) results of the non-correlated formulation are again in slightly better agreement with the correlated results. Although the approximate methods are again capable of predicting qualitatively correct source term distributions, their results are in greater error for this non-isothermal case, with up to 48 % and 42 % at $x = 1$ m in figures 6a and 6b, respectively.

Figure 6. Distributions of the predicted radiative source term along (x , 1 m, 0.15 m) and (x , 1 m, 3.55 m) for the non-isothermal case.

Figure 7 displays the calculated wall heat flux densities along $(2 \text{ m}, 1 \text{ m}, z)$ for the non-isothermal case. Similar to the results shown earlier, the grey-band model predicts slightly less accurate wall heat flux densities than the non-correlated formulation. Results of the grey-band model are in about 33 % error from the correlated results. Distributions of wall heat flux density along $(x, 1 \text{ m}, 0)$, on the inlet wall, and $(x, 1 \text{ m}, 4 \text{ m})$, on the exit wall, are shown in figure 8. Once again, results of the non-correlated formulation are slightly better than those of the grey-band model.

The approximate results are qualitatively correct and in about 30 % error from the correlated results at both the inlet wall (figure 8a) and the exit wall (figure 8b).

Figure 7. Distributions of the predicted wall heat flux density along $(2 \text{ m}, 1 \text{ m}, z)$ for the non-isothermal case.

3.2. Low-resolution spectral radiation intensity

Low-resolution spectral (narrow-band) radiation intensities, $\Delta\nu \times \bar{I}_\nu$, along three lines-of-sight for the isothermal case (the first case) are shown in figure 9. Figures 9a-c show the narrow-band radiation intensity viewed at $(1 \text{ m}, 1 \text{ m}, 0.25 \text{ m})$, $(1 \text{ m}, 1 \text{ m}, 1 \text{ m})$, and $(1 \text{ m}, 1 \text{ m}, 4 \text{ m})$, respectively, and along a direction defined by its direction cosine $(0.0990147, 0.0990147, 0.990147)$ (taken from the T_4 quadrature set). The direction of these three lines-of-sight is almost in parallel with the z -axis. Figure 9d is an enlarged portion of figure 9c showing details of the comparison in the wavenumber range between 500 and 3 500 cm^{-1} .

In figure 9a, results of the non-correlated formulation are identical to those of the correlated one. This is because the radiation beam under consideration only prop-

Figure 8. Distributions of the predicted wall heat flux density along $(x, 1 \text{ m}, 0)$ and $(x, 1 \text{ m}, 4 \text{ m})$ for the non-isothermal case.

agates across one computational grid and there is no correlation effect. This is also the case where the difference between the results of the non-correlated formulation and the grey-band method is the greatest. Such a large difference is attributed to the fact that the transmissivity used in the non-correlated formulation is based on the true distance travelled by the radiation beam, while in the grey-band approximation, the mean path-length of the computation grid, rather than the true distance, is used to evaluate the mean gas absorption coefficient. However, as the radiation beam travels across more computational grids, this effect becomes less significant, i.e. the low-resolution spectral intensities predicted by the non-correlated expression and the grey-band model are in much closer agreement with each other, as evidenced in figures 9b and 9c. In fact, results of the non-correlated formulation and the grey-band model are almost identical in figure 9c. Except for figure 9a, the narrow-band radiation intensity distributions predicted

Figure 9. Comparison of the low-resolution spectral intensities along three lines-of-sight having the same direction cosines (0.0990147, 0.0990147, 0.990147) for the isothermal case: (a) viewed at (1 m, 1 m, 0.25 m), (b) (1 m, 1 m, 1 m), (c) (1 m, 1 m, 4 m), (d) an enlarged portion of (c). The wavenumber range in figures 9a and b is 150 to 8 000 cm^{-1} .

Figure 10. Comparison of the low-resolution spectral intensities along three lines-of-sight having the same direction cosines (0.0990147, 0.0990147, 0.990147) for the non-isothermal case: (a) viewed at (1 m, 1 m, 0.4 m), (b) (1 m, 1 m, 2.6 m), (c) (1 m, 1 m, 4 m), (d) an enlarged portion of (c). The wavenumber range in figures 10a and b is 150 to 8 000 cm^{-1} .

by the non-correlated and the grey-band model follow the correlated results very faithfully. However, the errors of the approximate results could be as large as about 30 % in some spectral ranges (figure 9d). The approximate methods in general overpredict the spectral radiation intensity due to the neglect of the correlation term. This observation provides the explanation for the behaviour of the two approximation methods in the calculation of spectrally integrated quantities (figures 1–4).

Following Kim and Song [4], an average spectral error of an approximate method can be defined as

$$E = \frac{\sum_{\nu} |I_{\nu a} - I_{\nu e}|}{\sum_{\nu} I_{\nu e}} \quad (11)$$

where subscripts a and e denote approximate and exact solution, respectively. For the isothermal case, the average spectral errors of the non-correlated and the grey-band methods for the results shown in figure 9c are about 10 % (9.8 and 10.2 %, respectively), similar to the errors of wall heat flux densities shown in figures 3 and 4.

Figure 10 shows the low-resolution spectral radiation intensities along three lines-of-sight for the non-isothermal case. The ending points of these lines-of-sight are (1 m, 1 m, 0.4 m), (1 m, 1 m, 2.6 m) and (1 m, 1 m, 4 m), corresponding to figures 10a–c, respectively. These three beams have the same direction cosines, (0.0990147, 0.0990147, 0.990147), taken from the T_4 quadrature set. Figure 10d is an enlarged portion of the figure 10c to show details of the comparison in the spectral range of 1000 to 4500 cm^{-1} . Among the three lines-of-sight considered, the approximate methods predict very accurate low-resolution intensity for the shortest path-length (figure 10a). Note that the ending point of this beam is very close to the location of the peak gas temperature ($z = 0.375$ m) on the centreline. This is the result of the less important role of the correlation term for this short beam. For other two longer beams (figures 10b and 10c), the approximate methods are less accurate in the prediction of the low-resolution spectral intensity. It is believed that this is due to the increasing importance of the correlation term with increasing path-length in non-isothermal media. Although the approximate methods are still capable of predicting qualitatively correct spectral intensities in most of the spectral regions, they fail to do so in the spectral region between 1 250 and 1 750 cm^{-1} (figure 10d). Similar to the results of the isothermal case shown in figure 9, except figure 9a, the low-resolution spectral intensity predicted by the non-correlated formulation and the grey-band approximation are in good agreement with each other.

The average spectral errors of the non-correlated method and the grey-band model for the results shown in figure 10c are about 25 % (24.5 and 23.7 %, respectively).

CPU times required for each narrow-band calculation are compared in table I. A factor of 20 in CPU time

can be saved when the non-correlated formulation is used, compared to the correlated formulation. A direct comparison of CPU times between the grey-band approximation and the non-correlated expression is not meaningful, since two different solution methods were used. Nevertheless, the computational efficiency of the grey-band approximation can be clearly seen.

TABLE I
Comparison of CPU time for each narrow-band calculation.

Method	Correlated	Non-correlated	Grey-band
CPU (s)	1 044.88	56.64	1.44

3.3. Low-resolution spectral radiation intensity: parallel-plate enclosure

In order to provide a direct comparison of accuracy between the present two approximate methods and the narrow-band WSGG models investigated by Kim and Song [4] in the prediction of low-resolution spectral intensity, low-resolution spectral intensities were also calculated for the four one-dimensional problems studied by Kim and Song. Details of the four problems were given by Kim and Song and therefore are not repeated here. Twenty uniform gas layers were used in these one-dimensional calculations.

Average spectral errors of normal intensity at the right wall for various methods are compared in table II. In this table, models 2 and 4 are the two most accurate narrow-band WSGG models among the five models investigated by Kim and Song [4] who utilized five grey gases in each model. The two present approximate methods are less accurate than the two most accurate narrow-band WSGG models of Kim and Song. Compared to the performance of the two most accurate narrow-band WSGG models of Kim and Song, the present grey-band model does quite well considering the facts that five grey gases were used in the narrow-band WSGG models, while the grey-band model deals only with a single grey gas. It is worth pointing out that the decision on which model to use in the prediction of low-resolution spectral intensity in absorbing-emitting media is almost entirely determined by its accuracy, since in general such calculations require no iteration. However, this is no longer the case in the prediction of low-resolution spectral intensity in absorbing, emitting, and scattering media. For such calculations, the computational efficiency of the method to be used has to be taken into account, since iteration is inevitable due to the dependence of the in-scattering term upon the entire radiation field.

TABLE II
Average spectral errors, $\bar{E}$
(%) of normal radiation intensity at the right wall

Problem Definition	Uniform T	Parabolic T	Boundary-layer T	Parabolic concentration
Model 2	5.8	10.2	5.5	7.2
Model 4	5.8	10.3	5.4	7.0
Non-correlated	14.1	23.8	19.2	18.3
Grey-band	13.0	20.5	17.5	16.6

4. CONCLUSIONS

The accuracy of two approximate methods for implementing a narrow-band model, the non-correlated expression and the grey-band approximation, was further evaluated against the results obtained from the correlated formulation in a three-dimensional enclosure containing (1) isothermal pure water vapour, and (2) non-isothermal mixture of $\text{CO}_2/\text{H}_2\text{O}/\text{N}_2$. Evaluation was made for both the spectrally integrated quantities and the low-resolution spectral intensity.

Results of the two approximate methods are in good agreement with each other and in qualitative agreement with the correlated results for both the integrated quantities and the low-resolution intensity, except in some regions for the non-isothermal case. The present grey-band approximation offers the advantages of high computational efficiency, compatibility with the standard radiative transfer equation, and acceptable accuracy for engineering applications. The present grey-band model can be easily extended to problems with scattering and represent a good compromise between accuracy and computational efficiency for the prediction of low-resolution spectral intensity in three-dimensional absorbing, emitting, and scattering media.

Finally, we mention that a better strategy for predicting low-resolution spectral intensity in scattering media is to use a hybrid method developed from the grey-band approximation and a narrow-band weighted-sum-of-grey-gases model. The development of such a hybrid method is in progress.

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