Modeling failure risk in buried pipes using fuzzy Markov deterioration process

Kleiner, Y.; Sadiq, R.; Rajani, B. B.

Publisher's version / Version de l'éditeur:

ASCE International Conference on Pipeline Engineering and Construction [Proceedings], pp. 1-12, 2004-08-01

Modeling failure risk in buried pipes using fuzzy Markov deterioration process

Kleiner, Y.; Sadiq, R.; Rajani, B.

NRCC-46739

A version of this document is published in / Une version de ce document se trouve dans :
ASCE International Conference on Pipeline Engineering and Construction,
San Diego, CA., August 2-4, 2004, pp. 1-12

Modeling failure risk in buried pipes using fuzzy Markov deterioration process

[1*]Yehuda Kleiner, [2]Rehan Sadiq, and [1]Balvant Rajani

***ABSTRACT**: Numerous models have been proposed in the last two decades for the deterioration of buried pipes. The most prominent approach has been the Markovian deterioration processes (MDP), which requires that the condition of the deteriorating system be encoded as an ordinal condition state. This encoding is based on numerous distress indicators obtained possibly from direct and indirect observations, as well as from non-destructive tests. To date, few buried pipes have been inspected and their condition assessed. In addition, the encoding of distress indicators into condition states is inherently imprecise and involves subjective judgment. Furthermore, the consequences of failure for buried pipes are often difficult to quantify precisely due to lack of data.*

In this paper, a new approach is presented to model the deterioration of buried pipes using a fuzzy rule-based, non-homogeneous Markov process. This deterioration model yields possibility of failure at every point along the life of the pipe. The possibility of failure, expressed as a fuzzy number, is coupled with the failure consequence (also expressed as a fuzzy number) to obtain the failure risk as a function of the pipe age. The use of fuzzy sets and fuzzy techniques help to incorporate the inherent imprecision and subjectivity of the data, as well as to propagate these attributes throughout the model, yielding more realistic results. At the time of submission, adequate and sufficient data to validate the model were not available.

1 Introduction

Large buried pipes typically have low failure rates but when they fail the consequences can be quite severe. This low rate of failure, coupled with high cost of inspection and condition assessment, have contributed to the current situation where most municipalities lack the data necessary to model the deterioration rates of these assets and subsequently to make rational decisions regarding their renewal.

The condition assessment of a large buried pipe comprises two steps. The first step involves the inspection of the pipe using direct observation (visual, video) and/or non-destructive evaluation (NDE) techniques (radar, sonar, ultrasound, sound emissions, eddy currents, etc.), which reveal distress indicators. The second step involves the interpretation of these distress indicators to determine the condition state of the pipe. This interpretation process is dependent upon the inspection technique. The interpretation of the visual inspection results, although based on strict guideline, can often be influenced by subjective judgment. The interpretation of NDE results on the other hand, is often complex (at times proprietary) and imprecise.

[1] Senior Research Officer Institute for Research in Construction (IRC), National Research Council Canada (NRC)
[2] Research Officer Institute for Research in Construction, National Research Council Canada
* corresponding author

Managing the failure risk of large buried pipes requires a deterioration model to enable the forecast of the asset condition as well as its possibility of failure in the future. Significant research effort has been carried out in the last two decades to model infrastructure deterioration. The Markov deterioration process (MDP) is one approach that has gained prominence as exemplified by Madanat *et al*. (1997), Li *et al*. (1997), Abraham and Wirahadikusumah (1999), Wirahadikusumah *et al*. (2001), Mishalani and Madanat (2002), Kleiner (2001) and others. Examples of other types of statistical models include Lu and Madanat (1994), Ramia and Ali (1997), Flourentzou *et al*. (1999), Ariaratnam *et al*. (2001) and others.

In recent years increased research effort has been dedicated to the application of soft computing methods to assess infrastructure deterioration. Soft computing methods include techniques such as artificial neural network (ANN), genetic algorithms (GA), belief networks (BN), fuzzy sets and fuzzy techniques. Fuzzy techniques seem to be particularly suited to model the deterioration of infrastructure assets for which data are scarce and cause-effect knowledge is imprecise. Some examples from the literature of various applications of fuzzy techniques to infrastructure systems include: Chao and Cheng (1998) used a fuzzy-based pattern recognition model to diagnose cracks in reinforced concrete structures; Liang *et al*. (2001) developed a multiple-layer fuzzy method for concrete bridge health monitoring; Sadiq *et al*. (2004) employed a fuzzy-based technique for determining the soil corrosivity as a surrogate for breakage/corrosion rate in cast iron pipes..

2 Fuzzy sets

A fuzzy set describes the relationship between an uncertain quantity x and a membership function μ, which ranges between 0 and 1. A fuzzy set is an extension of the traditional set theory (in which x is either a member of set A or not) in that an x can be a member of set A with a certain degree of membership μ. Fuzzy techniques help address deficiencies inherent in binary logic and are useful in propagating uncertainties through models. A general definition of a fuzzy set is given by Dubois and Prade (1985): if x is a member of set A_i with a certain degree of membership $\mu_{A_i}(x)$, denoted as $A_i = \{(x, \mu_{A_i}(x))\}$, then A_i is a fuzzy set if x takes its value from the real numbers line and $\mu_{A_i}(x) \in [0, 1]$.

The proposed models use triangular fuzzy numbers (TFN), as these are often used for representing linguistic variables (Lee, 1996). To illustrate the concept, suppose that the age of a pipe is defined by five fuzzy subsets (or numbers), each representing an aging grade; A_1 = "*new*", A_2 = "*young*", A_3 "*medium*", A_4 = "*old*" and A_5 = "*very old*", as illustrated in Figure 1. The fuzzy subset A_3 "*medium*" for example, has a membership function such that for age x below 20 years or above 60 years the membership to "*medium*" is zero, and for age between 20 and 60 years the membership follows straight lines that form a triangle. Fuzzy set A comprises the collection of the five subsets (or numbers) A_i. The fuzzy subsets A_i are triangular fuzzy numbers that can be defined by three points representing the three vertices of the respective triangle, as shown in Figure 1.

In this example, it can be seen that for a pipe of age x = 50 years the membership values are $\mu_{A_3}(x)$ = 0.40, and $\mu_{A_4}(x)$ = 0.52 and zeros for $\mu_{A_1}(x)$ $\mu_{A_2}(x)$ and $\mu_{A_5}(x)$. The 5-tuple fuzzy set representing the buried pipe at age 50 can be written as the vector $A = (\mu_{A_1}(x), \mu_{A_2}(x), \mu_{A_3}(x), \mu_{A_4}(x), \mu_{A_5}(x))$ = (0, 0, 0.40, 0.52, 0), in which each element (tuple) depicts the pipe's membership value to the corresponding subset of aging grade (from *new* to *very old*).

There is a whole range of arithmetic operations defined for triangular fuzzy numbers Details of these arithmetic manipulations are described by Klir and Yuan (1995). The term "defuzzification" refers to a process to evaluate a crisp or point estimate of a fuzzy number. A defuzzified value is generally represented by a centroid, often determined using the center of area method (Yager, 1980).

subset	Qualitative scale	TFN representation
A_1	New	(0, 0, 20)
A_2	Young	(0, 20, 30)
A_3	Medium	(20, 40, 60)
A_4	Old	(40, 70, 100)
A_5	Very old	(70, 100, 100)

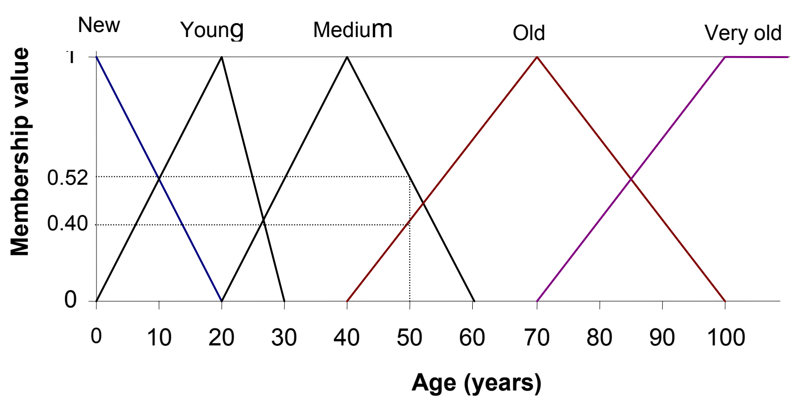


Figure 1. *Example of fuzzy sub-sets (numbers).*

3 The fuzzy rule-based algorithm

In fuzzy rule-based modeling, the relationships between variables are represented by means of fuzzy *if-then* rules of the form "*If* antecedent proposition *then* consequent proposition". The antecedent proposition is always a fuzzy proposition of the type "x is A" where x is a linguistic variable and A is a linguistic constant term. The proposition's truth-value (a real number between zero and 1) depends on the degree of similarity between x and A. This linguistic model (Mamdani, 1977) has the capacity to capture qualitative and highly uncertain knowledge in the form of *if-then* rules such as

$$\boldsymbol{R_i}: \textit{If } x \text{ is } A_j \textit{ then } y \text{ is } B_k; \quad i = 1, 2, \ldots, L; \; j = 1, 2, \ldots, M; \; k = 1, 2, \ldots, N \qquad (1)$$

where x is the input (antecedent) linguistic variable and A_j is an antecedent linguistic constant. (one of M in set A) Similarly, y is the output (consequent) linguistic variable and B_k is a consequent linguistic constant (one of N in set B). The values of x and y, and A_j and B_k are fuzzy sets defined in the domains of their respective base variables. The linguistic terms A_j and B_k are selected from sets of predefined terms, such as *small*, *medium*, *large*. The rule set (comprising L rules) and the sets A and B constitute the knowledge base of the linguistic model. Each rule is regarded as a fuzzy relation: $\boldsymbol{R_i}\ (X \times Y) \rightarrow [0, 1]$. This relation can be computed in two basic ways - by using fuzzy implications or fuzzy conjunctions, (Mamdani method), which were used in the proposed model. There are several steps involved in the Mamdani method, as described in Mamdani (1977) and in relevant textbooks, e.g., Yager and Filov (1994). The entire procedure can be summarized as

$$y = x \circ \boldsymbol{R} \qquad (2)$$

which means that if the rule set R is established, then for every input x, output y can be calculated (or inferred) using the appropriate operator "o". A fuzzy set can be "defuzzified", i.e. assigned a representative crisp value. There are several techniques in the literature for defuzzification, but the one used here is the most widely accepted technique known as the centroidal method (Yager, 1980).

The Mamdani inference algorithm can be extended to multiple inputs and single output (MISO):

$$\boldsymbol{R_i}: \textit{If } x_1 \text{ is } A_{1j} \textit{ and } x_2 \text{ is } A_{2j} \textit{ and} \ldots \textit{and } x_p \text{ is } A_{pj} \textit{ then } y \text{ is } B_k \qquad (3)$$

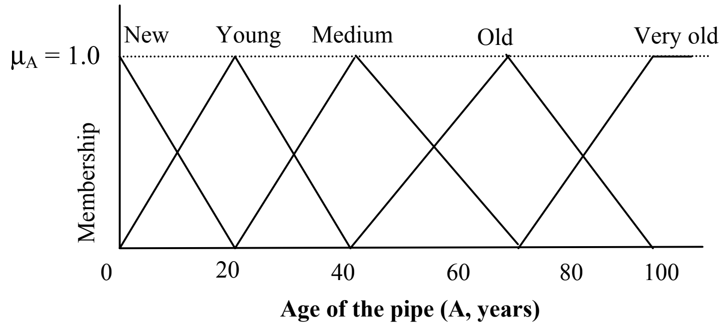
4 Fuzzy rule-based Markovian deterioration process (FR-MDP)

4.1 The knowledge base

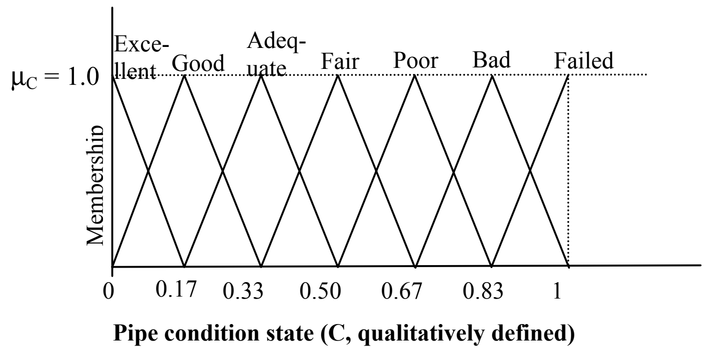
Figure 2 depicts the knowledge base for the proposed deterioration model. The age A of the pipe is partitioned into 5 levels (from *new* to *very old*), represented by triangular fuzzy subsets A_i (i = 1,2,…,5), with underlying units of years. Similarly, the condition C of the pipe is partitioned into 7 levels (from *excellent* to *failed*) represented by triangular fuzzy subsets C_i (i = 1,2,…,7). C is mapped onto an arbitrary unitless relative scale in the interval [0,1]. It should be noted that the *failed* state does not mean that collapse has already occurred (in which case the membership would be a clear unity), rather that it is imminent. The deterioration rate D' is partitioned into 5 levels (from *very slow* to *very fast*) represented by triangular fuzzy subsets D'_i (i = 1,2,…,5). D' is mapped onto a dynamic relative scale with underlying units of membership per

Knowledge-base

Age	Min	MLV	Max
New	0	0	20
Young	0	20	40
Medium	20	40	70
Old	40	70	100
Very old	70	100	100

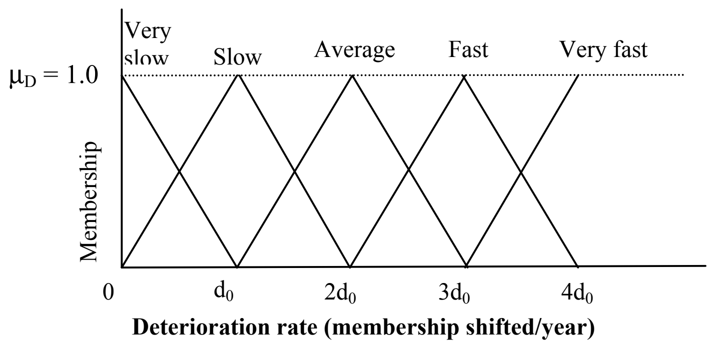


Condition state	Min	MLV	Max
Excellent	0	0	0.17
Good	0	0.17	0.33
Adequate	0.17	0.33	0.50
Fair	0.33	0.50	0.67
Poor	0.50	0.67	0.83
Bad	0.67	0.83	1
Failed	0.83	1	1



Deterioration rate	Min	MLV	Max
Very slow	0	0	d_0
Slow	0	d_0	$2d_0$
Average	d_0	$2d_0$	$3d_0$
Fast	$2d_0$	$3d_0$	$4d_0$
Very fast	$3d_0$	$4d_0$	$4d_0$

MLV - :most likely value



Fuzzy rule-set R_n

R_i = *If* pipe age (A) is **"*A*"** ***and*** pipe condition state (C) is **"*C*"** ***then*** deterioration rate (D) is **"*D*"'** (at time = *t*)

Pipe condition (C):	Excellent	Good	Adequate	Fair	Poor	Bad	Failed
Age (A): New	Slow	Average	Fast	Very fast	Very fast	Very fast	Very fast
Young	Slow	Average	Fast	Fast	Fast	Very fast	Very fast
Medium	Very slow	Slow	Average	Average	Fast	Fast	Very fast
Old	Very slow	Very slow	Slow	Slow	Average	Average	Fast
Very old	Very slow	Very slow	Very slow	Slow	Slow	Average	Average

Figure 2. *Fuzzy rule-base for the Markovian deterioration process (FR-MDP)*

year. The base deterioration rate parameter d_0, (Figure 2 – Deterioration rate chart) is found through regression as is later explained. The typical range of the deterioration scale will usually be between zero and 0.2 membership per year.

The table at the bottom of Figure 2 depicts the set of fuzzy rules R_D governing this model. For example, if the asset age is A= *young* and its condition is C = *fair* then its deterioration rate is D' = *fast*. The rule set R_D thus contains 35 fuzzy rules.

4.2 The deterioration process

The deterioration process is modeled as a "flow" of membership from one condition state to the next lower condition state. The deterioration in each time step comprises two steps. In the first step, the pipe age is fuzzified (mapped on A). The pipe's fuzzy condition state at time step (taken for convenience as a single year) t is C_t. The fuzzy deterioration at t, D'_t is computed using the Mamdani (1977) algorithm detailed earlier for the MISO model – equation (3) where A_t and C_t are the inputs, D'_t is the output and R_D is the fuzzy rule-set by which the fuzzy inferences are made.

$$D'_t = (A_t \wedge C_t) \circ R_D \quad (4)$$

D'_t, is a 5-tuple fuzzy set which is then defuzzified using the center of mass method. The defuzzified (crisp) value of the fuzzy deterioration D'_t is denoted by D_t. In the second step, the condition of the asset in the next time step C_{t+1} is calculated from its condition state in the current time step C_t and the (defuzzified) deterioration rate D_t obtained by rule-based algorithm in the current time step as follows

$$C_{t+1} = C_t \otimes D_t \quad (5)$$

where C_t is the condition at year t, D_t is the deterioration rate estimated by fuzzy rule set from A_t and C_t and $\otimes$ is an operator. The exact nature of this operator is discussed in detail in Kleiner et al. (2004). In essence it controls the "flow" of membership from one condition state to a more deteriorated condition state.

In traditional Markov deterioration models it is quite possible that at any time step t, significant memberships (or probabilities) in more that 3 conditions states can results. This outcome would be contrary to intuitive expert opinion. This situation is remedied by introducing threshold values, which restrict the membership "flow" from one condition state to the next. Figure 3 illustrates an example in which deterioration models with and without thresholds are compared. At $t = 40$ years for example, the condition state of the pipe in the model without threshold is approximately C_{40} = {0, 0.09, 0.16, 0.17, 0.20, 0.21, 0.17}, which means that the pipe has relatively significant membership value μ^{C_2}=0.09 to state 2 (*good*) and membership value μ^{C_7}=0.17 to state 7 (*failed*) simultaneously. This is of course un-realistic. In contrast, the model with threshold yields C_{40} = {0, 0.09, 0.36, 0.54, 0, 0, 0}, which is much more realistic. Membership to the *failed* state (state 7) at any given time t can be viewed as the possibility (not probability) of failure at that time.

In order to train the model to an existing asset one needs to know (or assume) the condition state of the asset immediately after installation, and at least one condition assessment at a later age *t*. The model is trained by minimizing the sum of square deviations between the observed and predicted membership values for time *t*. The

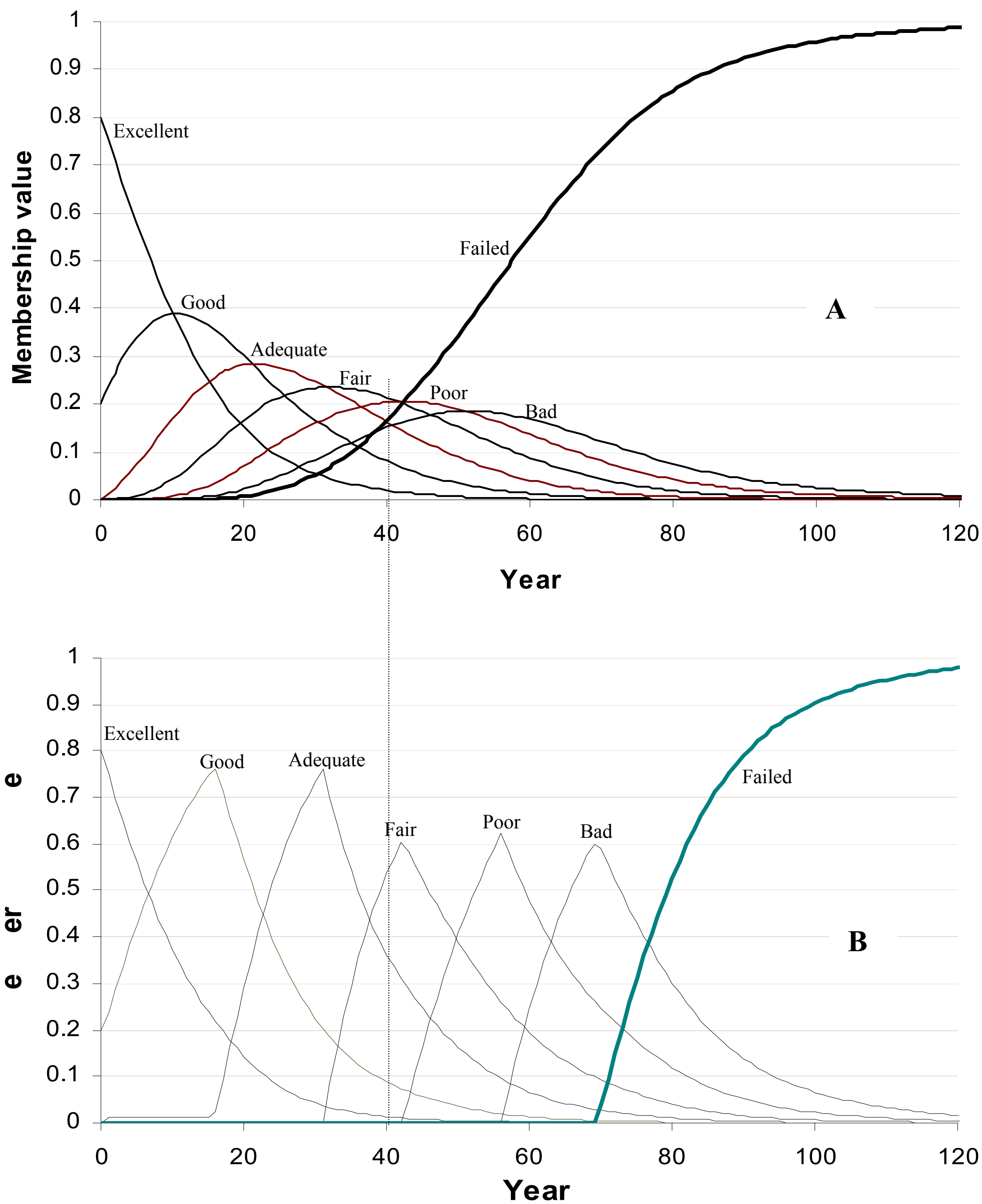


Figure 3. *Deterioration curves without (A) and with (B) membership thresholds.*

parameters that vary in the training process are those controlling the scale and shape of the deterioration fuzzy set, namely d_o and the aforementioned threshold values.

In order to validate the proposed model, one needs at least two consecutive observations of the asset condition. Further, these observations need to be reasonably distant (in time) from each other, to avoid errors due to small inconsistencies due to the subjective nature of the condition assessment of an asset. The first observation is needed to train the model and predict future deterioration, whereas the second observation is required to evaluate the prediction. Unfortunately, the data required for model validation were not available, making the validation of the model impossible.

4.3 Fuzzy possibility of failure

Fuzzy sets, such as TFNs are often interpreted as possibility distributions (in contrast to probability distribution) (Klir and Yuan, 1995). It follows that the membership value to the *failed* condition can be viewed as the possibility of failure. These membership values can be mapped onto a secondary fuzzy scale, comprising nine grades from *extremely low* to *extremely high*, as illustrated in Figure 4.

***Qualitative scale**	Min	MLV	Max
Extremely low	0	0	0.125
Very low	0	0.125	0.250
Quite low	0.125	0.250	0.375
Moderately low	0.250	0.375	0.500
Medium	0.375	0.500	0.625
Moderately high	0.500	0.625	0.750
Quite high	0.625	0.750	0.875
Very high	0.750	0.875	1.000
Extremely high	0.875	1.000	1.000

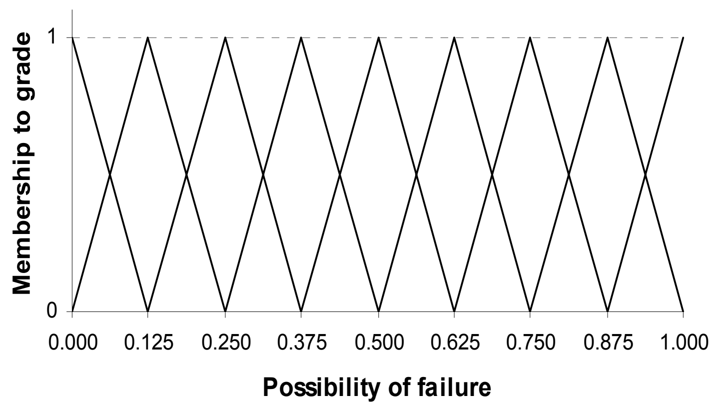


Figure 4. *Fuzzy possibility of failure*

5 Fuzzy rule-based risk

Lawrence (1976) defines risk it as *a measure of probability and severity of negative adverse effects*. When a complex system involves various contributory risk items with uncertain sources and magnitudes, it often cannot be treated with mathematical rigor during the initial or screening phase of decision-making (Lee, 1996). In the realm of buried pipes failures, not only is the likelihood of failure difficult to quantify, but failure consequences as well. Consequently, consequences of failure will be defined on a fuzzy qualitative nine-grade scale from *extremely low* to *extremely severe*.

A fuzzy rule-based MISO model is proposed for the risk analysis. The inputs to the model are the fuzzy possibility of failure, which is used in lieu of probability of failure, and fuzzy failure consequences. The output is risk level, which is also partitioned into 9 levels from *extremely low* to *extremely high*. The fuzzy rule-set (81 rules in total) is shown in Figure 5.

Failure Possibility	Failure Consequences: Extremely low	Very low	Quite low	Moderately low	Medium	Moderately severe	Quite severe	Very severe	Extremely severe
Extremely low	Extremely low	Extremely low	Very low	Very low	Quite low	Quite low	Moderately low	Moderately low	Medium
Very low	Extremely low	Very low	Very low	Quite low	Quite low	Moderately low	Medium	Medium	Moderately high
Quite low	Very low	Very low	Quite low	Quite low	Moderately low	Moderately low	Medium	Medium	Moderately high
Moderately low	Quite low	Quite low	Quite low	Moderately low	Moderately low	Medium	Medium	Moderately high	Quite high
Medium	Quite low	Moderately low	Moderately low	Moderately low	Medium	Medium	Moderately high	Moderately high	Quite high
Moderately high	Moderately low	Moderately low	Moderately low	Medium	Medium	Moderately high	Moderately high	Quite high	Very high
Quite high	Moderately low	Medium	Medium	Medium	Moderately high	Moderately high	Quite high	Quite high	Very high
Very high	Medium	Medium	Medium	Moderately high	Moderately high	Quite high	Very high	Very high	Extremely high
Extremely high	Medium	Moderately high	Moderately high	Quite high	Quite high	Very high	Very high	Extremely high	Extremely high

Figure 5. *Rule-base for fuzzy risk*

For example, immediately after installation a pipe is assumed to have been in a condition state represented by the fuzzy set $C_0 = (0.9, 0.1, 0, 0, 0, 0, 0)$ meaning 0.9 membership to *excellent* and 0.1 membership to *good*. At age 30 years an inspection and condition assessment was carried out and the pipe's condition was determined to be $C_{30} = (0, 0.2, 0.7, 0.1, 0, 0, 0)$ meaning 0.2 membership to *good*, 0.7 membership to *adequate* and 0.1 membership to *fair*. After a regression analysis, the resulting deterioration curves are as illustrated in Figure 6 below.

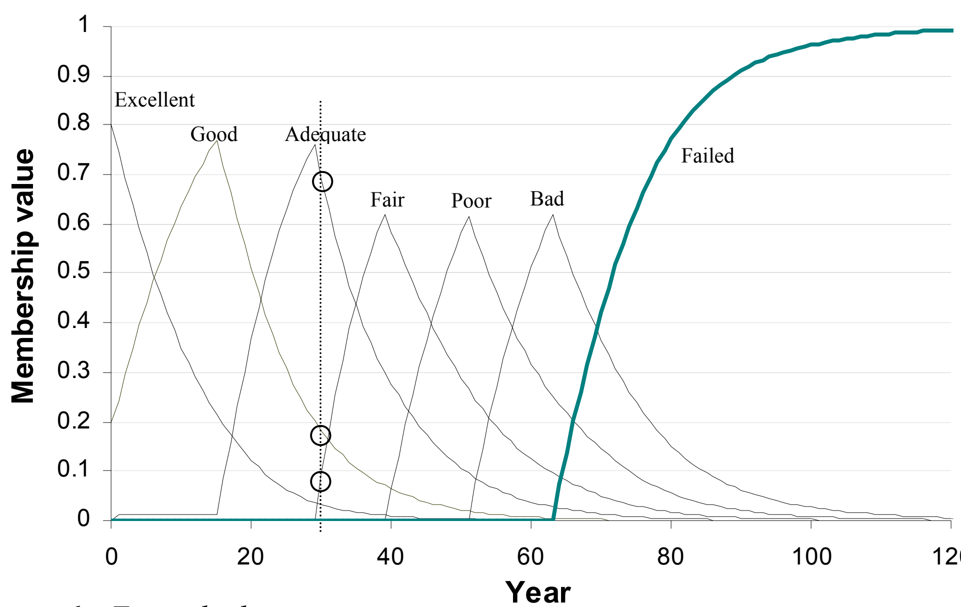


Figure 6. *Example deterioration curves*

For each year, *t,* in the life of the pipe, the membership to condition *failed* is re-mapped onto a fuzzy set depicting the possibility of failure (Figure 4).

Next, a pipe-failure consequence is arbitrarily (for this example) assumed to be represented by the fuzzy set $S = (0, 0, 0, 0, 0, 0.2, 0.5, 0.3, 0)$, meaning membership values of 0.2, 0.5 and 0.3 to fuzzy subsets *moderately severe*, *quite severe*, and *very severe* respectively. The resulting fuzzy risk curve is illustrated in Figure 7. The intensity of the gray levels represents the membership values to the respective risk levels. The black curve represents the defuzzified risk values. It can be seen that the defuzzified values do not always coincide with the highest membership values, which means that the fuzzy set representing risk at any year t is not always symmetrical about its mode.

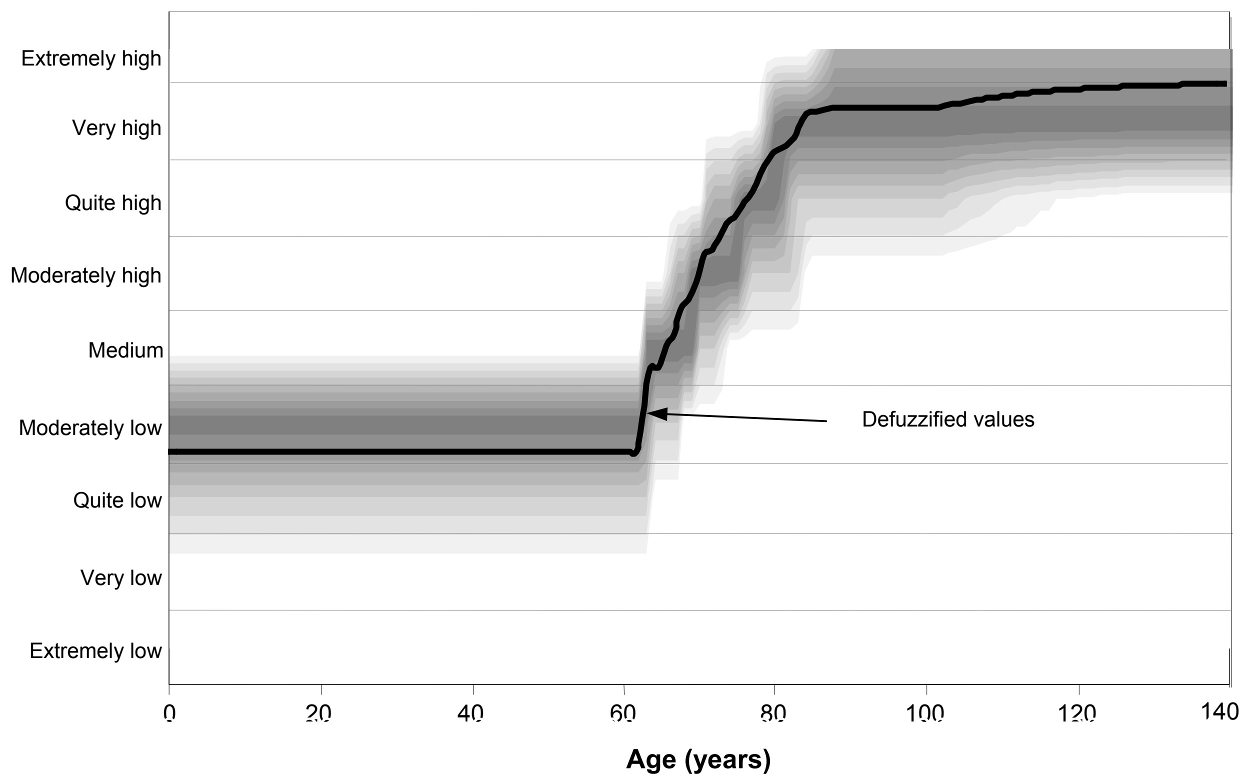


Figure 7. *Fuzzy risk levels over the life of a pipe*

4. Summary

The scarcity of data about the deterioration rates of buried infrastructure assets, coupled with the imprecise and often subjective nature of assessment of pipe condition merits the usage of fuzzy techniques in modeling the deterioration of these assets. The deterioration process is modeled as a fuzzy rule-based non-homogeneous Markov process applied at each time step in two stages. In the first stage, the deterioration rate at the specific time step is inferred from the asset age and condition state using a fuzzy rule-base algorithm. In the next stage, the condition state of the pipe is calculated from present condition state and deterioration rate. Essentially as the deterioration process progresses, the pipe gradually “flows” from higher membership in good condition states to higher membership in worse states. The process is formulated to mimic the reality in which a given asset at a given time cannot have significant membership values to more than two or three different (and consecutive) condition states.

The deterioration model is trained by non-linear regression in which the sum of square deviations between predicted and observed membership values is minimized. Data were not available to validate this model, but this should not deter water utilities from using it, as the model provides a framework for collecting the appropriate data, which would be required to validate any model. The model can be used to predict the future deterioration rate of the asset, subject to some judgment-based assumptions.

Once deterioration curves are obtained, the membership value to the *failed* state is viewed as the possibility of failure and is mapped onto a secondary fuzzy scale with nine failure possibility grades ranging from *extremely low* to *extremely high*. The consequences of pipe failure are defined on a fuzzy scale with nine intensity grades ranging from *extremely low* to *extremely severe.* The level of risk, which is also defined on a nine-grade fuzzy scale from *extremely low* to *extremely high*, can then be determined (inferred) based on a fuzzy rule base.

Acknowledgement

This paper presents interim results of an on-going research project, which is co-sponsored by the American Water Works Association Research Foundation (AwwaRF), the National Research Council of Canada (NRC) and water utilities from the United States, Canada and Australia.

References

Abraham, D.M., and Wirahadikusumah, R. 1999. Development of prediction model for sewer deterioration, *Proceedings of the 8th Conference Durability of Building Materials and Components*, Edited by M.A. Lacasse and D.J. Vanier, NRC, pp. 1257-1267, Vancouver.

Ariaratnam, S.T., A. El-Assaly, and Y. Yang. 2001. Assessment of infrastructure needs using logistic models, *Journal of Infrastructure Systems, ASCE*, 7(4): 160-165.

Chao, C-J., and Cheng, F.P. 1998. Fuzzy pattern recognition model for diagnosing cracks in RC structures, *Journal of Computing in Civil Engineering, ASCE*, 12(2): 111-119.

Dubois, D., and Prade, H. 1985. Evidence measures based on fuzzy information, *Automatica*, 21(5): 547-562.

Flourentzou, F., E. Brandt, and C. Wetzel. 1999. MEDIC – a method for predicting residual service life and refurbishment investment budgets, *Proceedings of the 8th conference Durability of Building Materials and Components*, Edited by M.A. Lacasse and D.J. Vanier, IRC, NRC, pp. 1280-1288, Vancouver.

Kleiner, Y. 2001. Scheduling inspection and renewal of large infrastructure assets, *Journal of Infrastructure Systems*, *ASCE*, 7(4): 136-143.

Kleiner, Y. Sadiq, R., and Rajani, B.B. 2004. Modeling the deterioration of buried infrastructure as a fuzzy Markov process, Submitted to *Journal of Infrastructure Systems*, *ASCE*.

Klir, G.J., and Yuan, B. 1995. *Fuzzy sets and fuzzy logic - theory and applications*, Prentice- Hall, Inc., Englewood Cliffs, NJ, USA.

Lawrence, W.W. 1976. *Of acceptable risk*, William Kaufmann, Los Altos, CA.

Lee, H.-M. 1996. Applying fuzzy set theory to evaluate the rate of aggregative risk in software development, *Fuzzy Sets and Systems*, 79: 323-336.

Li, N., Haas, L.R., and Xie, W-C. 1997. Development of a new asphalt pavement performance prediction model, *Canadian Journal of Civil Engineering*, 24: 547-559.

Liang, M.T., Wu, J.H., and Liang, C.H. 2001. Multiple layer fuzzy evaluation for existing reinforced concrete bridges, *Journal of Infrastructure Systems, ASCE,* 7(4): 144-159.

Lu, Y., and S.M. Madanat. 1994. Bayesian updating of infrastructure deterioration models, *Transportation Research Record*, 1442: 110-114.

Madanat, S.M., Karlaftis, M.G., and McCarthy, P.S. 1997. Probabilistic infrastructure deterioration models with panel data, *Journal of Infrastructure Systems, ASCE*, 3(1): 4-9.

Mishalani, R.G., and S.M. Madanat. 2002. Computation of infrastructure transition probabilities using stochastic duration models, *Journal of Infrastructure Systems, ASCE*, 8(4): 139-148.

Madanat, S.M., Mishalani, R., and Wan Ibrahim, W.H. 1995. Estimation of infrastructure transition probabilities from condition rating data, *Journal of Infrastructure Systems, ASCE*, 1(2): 120-125.

Mamdani, E.H. 1977. Application of fuzzy logic to approximate reasoning using linguistic systems, *Fuzzy Sets and Systems*, 26: 1182-1191.

Ramia, A.P., and N. Ali. 1997. Bayesian methodologies for evaluating rutting in Nova Scotia’s special B asphalt concrete overlays, *Canadian Journal of Civil Engineering*, 24(4): 1-11.

Sadiq, R., Rajani, B.B. and Kleiner, Y. 2004. A fuzzy based method of soil corrosivity evaluation for predicting water main deterioration, Submitted to *Journal of Infrastructure Systems*, *ASCE.*

Wirahadikusumah, R., Abraham, D., and Isely, T. 2001. Challenging issues in modeling deterioration of combined sewers”, *Journal of Infrastructure Systems, ASCE*, 7(2): 77-84.

Yager, R.R. 1980. A general class of fuzzy connectives, *Fuzzy Sets and Systems*, 4: 235-242.

Yager, R.R., and Filev, D.P. 1994. *Essentials of fuzzy modeling and control*, John Wiley & Sons, Inc., NY.