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CREEP MODEL OF ICE FOR MONOTONICALLY INCREASING STRESS

by **N.K. Sinha**

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RÉSUMÉ

Une méthode d'intégration numérique pour prédire les profils de déformation correspondant à des contraintes prolongées et établir par la suite la relation contrainte-déformation, a été mise au point pour des matériaux polycristallins comme la glace, à de hautes températures homologues. La méthode est basée sur une équation généralisée du fluage incorporant l'effet de la dimension des grains. On peut ainsi prévoir les déformations élastiques, élastiques retardées, visqueuses et permanentes. La théorie a été éprouvée avec succès lors d'essais de résistance de la glace à des températures variant entre -5 et -30°C . L'augmentation monotone du taux de déformation observée durant les essais à une vitesse constante d'application de la contrainte peut maintenant être expliquée analytiquement.

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CREEP MODEL OF ICE FOR MONOTONICALLY INCREASING STRESS

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ABSTRACT

A numerical integration method of predicting the strain path corresponding to a given stress history, and hence the stress-strain relation, has been developed for polycrystalline materials like ice at high homologous temperatures. The method is based on a generalized creep equation incorporating the grain size effect and predicts strain in terms of elastic, delayed-elastic, and viscous or permanent strains. The theory has been tested successfully during strength tests of ice in the temperature range of -5 to -30°C . The monotonous increase in strain rate observed during testing under conditions of constant cross-head displacement rate can now be explained analytically.

INTRODUCTION

The development of a phenomenological, non-linear, viscoelastic equation for polycrystalline ice of a given grain size has made it possible to explain, in a general way, inconsistencies in the results of earlier creep investigations of ice (Sinha, 1978a, b). The model was modified analytically to incorporate the effect of grain size, and a generalized constitutive equation was developed by comparing the creep response of ice with that of metals, alloys and other polycrystalline materials at high homologous temperatures (Sinha, 1979a). This allowed several other peculiarities of the creep behaviour of ice to be clarified (Sinha, 1979b).

It is the main purpose of the present paper to present the principles involved in utilizing the generalized creep equation for predicting high temperature

response of polycrystalline materials such as ice subjected to loading conditions of variable stress. The meaning of "ice-like" will be clarified. Experimental results obtained during strength testing (Sinha, 1981) of columnar-grained ice have been used to test the theory.

The author earlier demonstrated that specimen strain rates were not constant during tests under constant cross-head rates (Sinha, 1981) and that consequently the strengths and stress-strain results were not representative of truly constant strain rate conditions; this investigation refuted the popular assumption of a constant strain rate corresponding to a constant cross-head displacement rate. The secondary purpose of the present paper is to establish whether the experimental observations can be described analytically or not.

Generalized viscoelastic equation

Creep deformation in polycrystalline materials at high temperatures can be expressed generally by

$$\epsilon_t = \epsilon_e + \epsilon_d + \epsilon_v \quad (1)$$

where ϵ_e is the pure elastic deformation, ϵ_d is the recoverable, delayed elastic strain, and ϵ_v is the viscous or permanent deformation. The term "viscous" will be used in a general sense to describe flow leading to permanent deformation. The term "plastic" is avoided here because it is commonly used to lump permanent and delayed elastic deformation.

For conditions where grain boundary diffusional processes do not play a dominating role and where the microstructure has not deteriorated by the for-

mation of internal voids and cracks, creep in previously undeformed, pure, randomly oriented, polycrystalline materials may be expressed in terms of stress, σ , and time, t , for a material with an average grain diameter of d at a given temperature, T , by (Sinha, 1979a)

$$\epsilon_t = \frac{\sigma}{E} + c_1 \left(\frac{d_1}{d} \right) \left(\frac{\sigma}{E} \right)^s \{ 1 - \exp[-(a_T t)^b] \} + \dot{\epsilon}_{v_1} \left(\frac{\sigma}{\sigma_1} \right)^n t \quad (2)$$

In this equation the three terms correspond to the three successive terms in eqn. (1).

In eqn. (2), E is Young's modulus; a_T , b , c_1 , n and s are constants; d_1 is the unit of grain diameter; and $\dot{\epsilon}_{v_1}$ is the viscous strain rate for unit stress σ_1 . The constant a_T is the inverse of relaxation time at temperature T ; n is the stress exponent for viscous flow; and s is the stress exponent for delayed elasticity.

Both $\dot{\epsilon}_{v_1}$ and a_T are dependent on temperature:

$$\dot{\epsilon}_{v_1}(T_1) = \frac{\dot{\epsilon}_{v_1}(T_2)}{S_{1,2}} \quad (3)$$

and

$$a_T(T_1) = \frac{a_T(T_2)}{S_{1,2}} \quad (4)$$

where $S_{1,2}$ is a shift function (Sinha, 1978a), given by

$$S_{1,2} = \exp \left[\frac{Q}{R} \left(\frac{1}{T_1} - \frac{1}{T_2} \right) \right] \quad (5)$$

where Q and R are the activation energy and gas constant, respectively; T_1 and T_2 are temperatures in Kelvin.

PRINCIPLE OF CALCULATIONS

Before proceeding it is necessary to illustrate the principles involved in the application of the model to a two-stage uniaxial creep loading at constant temperature. In the first stage, a load $\sigma = \sigma_1$ is applied instantaneously at $t = 0$ and then held constant thereafter (Fig. 1). After a creep time, $t = t_1$, the stress is suddenly increased by an amount σ_2 to $\sigma = \sigma_1 + \sigma_2$. The question is, what would be the strain at time $t > t_1$?

Fig. 1. Two-stage creep loading and strain component. Results are calculated for $\sigma_1 = \sigma_2 = 1 \text{ MN m}^{-2}$ and $t_1 = 600 \text{ s}$ for $d = 2 \text{ mm}$ at $T = 263 \text{ K}$.

(a) Pure elastic strain is dependent on stress according to Hooke's law, as indicated by the first term in eqn. (2). Hence,

$$\epsilon_e(\sigma_1 + \sigma_2, t > t_1) = \epsilon_e(\sigma_1) + \epsilon_e(\sigma_2) \quad (6a)$$

or

$$\epsilon_e = \frac{\sigma_1}{E} + \frac{\sigma_2}{E} = \frac{\sigma_1 + \sigma_2}{E} \quad (6b)$$

(b) Delayed elastic strain and its rate of change with time are dependent on both stress and time, according to the second term in eqn. (2). These quantities are therefore dependent on the history of loading. This is difficult to handle, in general, and will be discussed elsewhere in detail. For the special case where $s = 1$, the second term in eqn. (2) and its first derivative with respect to time become linearly dependent on stress. In this case the principle of superposition, similar to that usually associated with a standard linear solid (Finnie, 1959; Lee, 1962; Hart et al., 1975), can be applied. The strain at $t > t_1$, will then be given as the sum of the strain components at t due to σ_1 applied at $t = 0$

and due to σ_2 applied at $t = t_1$ (Fig. 1). Thus

$$\epsilon_d(\sigma_1 + \sigma_2, t > t_1) = \epsilon_d(\sigma_1, t) + \epsilon_d(\sigma_2, t - t_1) \quad (7a)$$

Following the second term of eqn. (2), with $s = 1$, this reduces to

$$\begin{aligned} \epsilon_d = c_1 \left(\frac{d_1}{d} \right) \left(\frac{\sigma_1}{E} \right) \{ 1 - \exp [- (a_T t)^b] \} \\ + c_1 \left(\frac{d_1}{d} \right) \left(\frac{\sigma_2}{E} \right) [1 - \exp [- [a_T(t - t_1)]^b] \} \end{aligned} \quad (7b)$$

(c) Viscous strain, according to the third term in eqn. (2), is stress- and time-dependent, but strain rate is independent of time under constant stress. This type of flow under the present loading conditions, considered at $t > t_1$, will therefore be given as the sum of the strain (Fig. 1) produced by σ_1 applied for t_1 and the strain produced by $\sigma_1 + \sigma_2$ applied for times greater than t_1 . It can be stated mathematically as

$$\epsilon_v(\sigma_1 + \sigma_2, t > t_1) = \epsilon_v(\sigma_1, t_1) + \epsilon_v(\sigma_1 + \sigma_2, t - t_1) \quad (8a)$$

This gives, according to the third term in eqn. (2),

$$\epsilon_v = \dot{\epsilon}_{v_1} \left(\frac{\sigma_1}{\sigma^1} \right)^n t_1 + \dot{\epsilon}_{v_1} \left(\frac{\sigma_1 + \sigma_2}{\sigma^1} \right)^n (t - t_1). \quad (8b)$$

The total deformation will be the sum of the components given by eqns. (6b), (7b) and (8b) as shown in Fig. 1.

It must be noted that the principle enunciated in this analysis of delayed elasticity is applicable only for the class of polycrystalline material that exhibits linear stress dependency ($s = 1$) in its delayed elastic

TABLE 1

Creep parameters for ice obtained independently from earlier creep experiments and analyses (Sinha 1978a, 1979a)

E	$= 9.5 \text{ GN m}^{-2}$
Q	$= 67 \text{ kJ/mole (16 kcal/mole)}$
c_1	$= 9$
d_1	$= 1 \text{ mm}$
s	$= 1$
n	$= 3$
b	$= 0.34$
$a_T(T = 263 \text{ K})$	$= 2.5 \times 10^{-4} \text{ s}^{-1}$
$\dot{\epsilon}_{v_1}$	$= 1.76 \times 10^{-7} \text{ s}^{-1}; \sigma^1 = 1 \text{ MN m}^{-2}, T = 263 \text{ K}$

response. Material constants determined for ice (Sinha, 1978a, b) and given in Table 1 indicates that ice belongs to this class of material. As these principles will be applied next to the case of varying stress, the following analyses are applicable to "ice-like" materials.

Variable stress loading

Most strength tests are performed by increasing the stress until failure occurs. The usual continuous stress path can be presented in terms of a stepped stress path in the fashion of a series of creep loadings at equal intervals of time, Δt . Suppose $\Delta\sigma_1, \Delta\sigma_2, \dots, \Delta\sigma_N, \Delta\sigma_{N+1}$ are the incremental increases in stress corresponding to times $0, \Delta t, 2\Delta t, \dots, (N-1)\Delta t, N\Delta t$ such that $\Delta\sigma_1$ represents the average stress in the first interval of time, and $\Delta\sigma_1 + \Delta\sigma_2$ in the second interval, etc.

The stress step $\Delta\sigma_1$ applied at $t = 0$ will produce an instantaneous elastic strain of $\Delta\sigma_1/E$ at $t = 0^+$ and contributions from delayed elastic and viscous flow will be negligible. In accordance with the principles of (a), (b) and (c),

$$t = 0^+; \quad \epsilon_t = \frac{\Delta\sigma_1}{E} + 0^+ + 0^+ \quad (9)$$

At the beginning of the second interval of time, $t = 1\Delta t^+$, there will be elastic strain due to total stress of $\Delta\sigma_1 + \Delta\sigma_2$. Delayed elastic strain will be the strain produced by $\Delta\sigma_1$ applied for $1\Delta t$; there will be negligible contribution at this time from $\Delta\sigma_2$ because this stress applied for 0^+ time introduces negligible delayed elastic strain. Viscous strain at this time will be the contribution of $\Delta\sigma_1$ applied for Δt with no, or negligible, contribution from $\Delta\sigma_1 + \Delta\sigma_2$. Thus, at $t = 1\Delta t^+$

$$\epsilon_e = \frac{\Delta\sigma_1 + \Delta\sigma_2}{E} \quad (10a)$$

$$\epsilon_d = c_1 \left(\frac{d_1}{d} \right) \left(\frac{\Delta\sigma_1}{E} \right) \{ 1 - \exp [- (a_T 1\Delta t)^b] \} + 0^+ \quad (10b)$$

$$\epsilon_v = \dot{\epsilon}_{v_1} \left(\frac{\Delta\sigma_1}{\sigma^1} \right)^n \Delta t + 0^+ \quad (10c)$$

These methods of analysis can be applied for any other times. In general, the elastic components at

$t = N\Delta t^*$ are given by

$$\epsilon_e = \frac{\Delta\sigma_1}{E} + \frac{\Delta\sigma_2}{E} + \dots + \frac{\Delta\sigma_{N+1}}{E} = \frac{1}{E} \sum_{i=1}^{N+1} \Delta\sigma_i \quad (11a)$$

The corresponding delayed elastic strain is given as

$$\begin{aligned} \epsilon_d = & c_1 \left(\frac{d_1}{d} \right) \left(\frac{\Delta\sigma_1}{E} \right) \{1 - \exp[-(a_T N \Delta t)^b]\} \\ & + c_1 \left(\frac{d_1}{d} \right) \left(\frac{\Delta\sigma_2}{E} \right) \{1 - \exp[-(a_T [N-1] \Delta t)^b]\} \\ & + \dots \\ & + c_1 \left(\frac{d_1}{d} \right) \left(\frac{\Delta\sigma_N}{E} \right) \{1 - \exp[-(a_T 1 \Delta t)^b]\} \\ & + c_1 \left(\frac{d_1}{d} \right) \left(\frac{\Delta\sigma_{N+1}}{E} \right) \{1 - \exp[-(a_T 0 \Delta t)^b]\} \\ = & \frac{c_1}{E} \left(\frac{d_1}{d} \right) \sum_{i=1}^{N+1} \Delta\sigma_i \{1 - \exp[-(a_T [N+1-i] \Delta t)^b]\} \end{aligned} \quad (11b)$$

and the viscous strain by

$$\begin{aligned} \epsilon_v = \dot{\epsilon}_{v_1} \Delta t \left[\left(\frac{\Delta\sigma_1}{\sigma^1} \right)^n + \left(\frac{\Delta\sigma_1 + \Delta\sigma_2}{\sigma^1} \right)^n + \dots \right. \\ \left. + \left(\frac{\Delta\sigma_1 + \Delta\sigma_2 + \dots + \Delta\sigma_N}{\sigma^1} \right)^n \right] \\ = \dot{\epsilon}_{v_1} \Delta t \sum_{i=1}^N \left(\frac{1}{\sigma^1} \sum_{j=1}^i \Delta\sigma_j \right)^n \end{aligned} \quad (11c)$$

The total strain will be given by the summation of the components (11a), (11b) and (11c). Actual loading conditions of continuous increase in load with time will be approached as Δt is decreased and therefore N is increased.

CONSTANT STRESS RATE

In the special case where stress rate is constant at $\dot{\sigma}$, equal intervals of time, Δt , result in equal stress steps

$$\Delta\sigma = \dot{\sigma}\Delta t \quad (12)$$

and the stress at the end of the N th period of time is

$$\sigma = N\Delta\sigma = N\dot{\sigma}\Delta t \quad (13)$$

Correspondingly, the elastic strain for $t = N\Delta t^*$

$$\epsilon_e = \frac{N\Delta\sigma}{E} \quad (14a)$$

The average stress in the first interval will be $\Delta\sigma/2$ and consequently that in the N th interval will be $(N - \frac{1}{2})\Delta\sigma$. It is, however, simple to replace $\Delta\sigma_i$ in eqn. (11b) by $\Delta\sigma$ to get

$$\epsilon_d = \frac{c_1}{E} \left(\frac{d_1}{d} \right) \sum_{i=1}^{N+1} \Delta\sigma \{1 - \exp[-(a_T [N+1-i] \Delta t)^b]\} \quad (14b)$$

This does not introduce any significant error in the calculations if N is large (as will be illustrated later). Similarly, eqn. (11c) gives

$$\epsilon_v = \dot{\epsilon}_{v_1} \Delta t \sum_{i=1}^N \left(\frac{i\Delta\sigma}{\sigma^1} \right)^n \quad (14c)$$

The elastic strain of eqn. (14a) in its integral form can simply be given as

$$\epsilon_e = \sigma/E \quad (15a)$$

where σ corresponds to the loading of $t = \sigma/\dot{\sigma}$.

A simple closed-form integral solution appropriate to eqn. (14b) for the delayed elastic term has not been obtained so far, and calculations have to be made by the numerical integration method. Delayed elastic strain, ϵ_d , at any time given by this equation, however, rapidly converges as Δt is decreased and thereby makes the calculations relatively simple. This will be clarified later.

The viscous term, eqn. (14c), can be integrated by substituting $d\sigma/\dot{\sigma}$ for Δt and σ for $i\Delta\sigma$, and integrating over the stress range of zero to the final stress, giving

$$\epsilon_v = \dot{\epsilon}_{v_1} \int_0^\sigma \left(\frac{\sigma}{\sigma^1} \right)^n \frac{d\sigma}{\dot{\sigma}} = \frac{\dot{\epsilon}_{v_1} \sigma^1}{\dot{\sigma}(n+1)} \left(\frac{\sigma}{\sigma^1} \right)^{n+1} \quad (15b)$$

Equations (15a) and (15b) can now be combined with eqn. (14b) for slightly better computed results, for a given Δt value chosen to calculate the delayed elastic strain, than the combination of eqns. (14a), (14b) and (14c).

EXPERIMENT

A conventional screw-driven test machine capable of delivering constant displacement rates was used to investigate the rate sensitivity of uniaxial strength of columnar-grained, S-2 ice of average cross-sectional grain diameter of 4 to 5 mm at -10°C (Sinha, 1981). It was observed that the specimen strain rates were far from constant during the tests for a given nominal strain rate, $\dot{\epsilon}_n = \dot{x}/l$, corresponding to a cross-head rate $\dot{x}$ and specimen length l .

In spite of the observed constancy in cross-head displacement rates, strain rates in the specimens monotonously increased with time and approached the nominal strain rate only after reaching maximum stress for upper-yield type of failure. The initial strain rates were about an order of magnitude less than the subsequent maximum strain rate. The peak strain rate in the specimen, $\dot{\epsilon}_p$, estimated from a narrow range

of strain--time data near yield point or after maximum stress was reached, compared well with the corresponding $\dot{\epsilon}_n$. These observations indicated that the testing machine was able to impose the specified deformation rate on the specimen only when the specimen offered no more increase in resistance with further increase in deformation. The results, therefore, could not be construed (Sinha, 1981) as constant strain rate, although this is customarily assumed.

Analysis showed that the actual loading history comprised a nearly constant stress rate up to about 90% of yield stress, followed thereafter by a path leading to a constant strain rate. An example of such a test result is shown in Fig. 2, which gives stress and strain history as well as the stress--strain diagram for an imposed nominal strain rate of $5 \times 10^{-5} \text{ s}^{-1}$. In this case a constant stress rate of $7.8 \times 10^{-2} \text{ MN m}^{-2} \text{ s}^{-1}$ describes the load path up to about 90% of the maximum stress.

COMPARISON WITH THEORY

Computations using eqns. (15a) and (15b) for the elastic and viscous strains, respectively, and eqn. (14b) for delayed elastic strain for a constant stress rate of $7.8 \times 10^{-2} \text{ MN m}^{-2} \text{ s}^{-1}$ are shown in Fig. 3. Rapid convergence in the calculated delayed-elastic strain with decrease in Δt is also shown. A solid line for the delayed-elastic strain was calculated using $\Delta t = 0.3 \text{ s}$ in eqn. (14b), whereas the broken line shows the results for $\Delta t = 3 \text{ s}$. Calculations were made using the experimentally obtained average grain diameter (Sinha, 1981), $d = 4.5 \text{ mm}$, $T = 263 \text{ K}$, and previously determined creep parameters (Sinha, 1978a; 1979a) given in Table 1.

These theoretical results for total strain are compared with experimental observations in Fig. 2. As stress rate deviated (significantly) from the constant rate above a stress of about 4.5 MN m^{-2} , or beyond a loading time of about 60 s, theoretical results should not be applied beyond these stress or time limits. Figure 2 shows excellent agreement between theory and experiment within these stress and time limits. Beyond about 60 s the good agreement is only apparent, as may be seen when the theoretical stress--strain curve is compared with the experimental one.

Fig. 2. Stress and strain history and stress--strain results on columnar-grained ice of average grain diameter of 4.5 mm at -10°C for nominal strain rate of $5 \times 10^{-5} \text{ s}^{-1}$.

Fig. 3. Theoretical elastic, delayed-elastic, viscous strains and hence total strain for a loading condition of constant stress rate of $7.8 \times 10^{-2} \text{ MN m}^{-2} \text{ s}^{-1}$ for ice with average grain diameter of 4.5 mm at -10°C .

The time dependence of strain calculated from the actual stress history for the example under consideration is also shown in Fig. 2. Computations were made using eqns. (11a), (11b) and (11c), Table 1, and measured stress steps at $\Delta t = 1 \text{ s}$, keeping grain size the same as before. The results show better agreement with the overall experimental observations than do the previous calculations. This is particularly noticeable in the stress-strain diagram. The time dependence of the strain shows, however, increasing deviation in the latter calculation. This failure to predict the proper time dependence of the strain, as the test progresses towards yield, indicates that the theory is no longer applicable — probably because of breakdown of the structure as a result of formation of cracks.

The basic constitutive eqn. (2) does not take into account the contribution of microcracking to total strain. The present theory is expected to predict

less than the actual strain if cracking occurs. Internal cracks started to develop in the illustrated case at a loading time of about 21 s, or at a stress level of 1.6 MN m^{-2} (Fig. 2). Examination of the specimen after unloading at 84 s disclosed 470 long narrow cracks in the central $10 \times 10 \text{ cm}$ area of the $5 \times 10 \times 25 \text{ cm}$ specimen. These cracks were about 0.5 to 5 mm wide, 5 mm or more in length, and fractions of 1 mm in thickness. They occurred mainly at grain boundaries (Sinha, 1981) and many were as long as the length of the columnar grains. It was realized that systematic studies of the formation of cracks and their influence on rheological response is required for further improvement of the theory. In spite of these limitations the theoretical strain of 13.6×10^{-4} , predicted at an upper yield point of 5.04 MN m^{-2} reached at 77 s, is only about 10% less than the corresponding experimental strain of 15.2×10^{-4} . Moreover, the theoretical ($1.8 \times 10^{-5} \text{ s}^{-1}$) and experimental ($2.0 \times 10^{-5} \text{ s}^{-1}$) average strain rates to peak stress are in reasonable agreement, although both are quite different from the nominal strain rate of $5 \times 10^{-5} \text{ s}^{-1}$. This example shows that the agreement between theory and experiment is acceptable even for conditions of relatively severe cracking activity*.

Further comparisons of theory and experiment are shown in Figs. 4 and 5 for tests conducted at $\dot{\epsilon}_n = 1.3 \times 10^{-4} \text{ s}^{-1}$ and $5 \times 10^{-6} \text{ s}^{-1}$, respectively. The rates are different from the loading rate used in Fig. 2 and as a result cracking activity also varied greatly. In the former, 700 cracks were counted in the central $10 \times 10 \text{ cm}$ area after unloading, compared with about 70 in the latter. The examples in support of the theory are only three of several experiments carried out during the test program reported by Sinha (1981). It is not possible to show all the results in the limited space available, but it is acknowledged that the discussion will not be complete without a brief, overall analysis.

The maximum stress, σ_f , at upper yield or time of premature splitting type failure is presented in Fig. 6 as a function of average strain rate, $\dot{\epsilon}_{af}$, defined as ϵ_f/t_f where ϵ_f is the strain at maximum stress and

*It may be appropriate here to mention that analyses presented by Sinha (1982b) strongly indicate that the present theory can be used to predict the onset of cracking activity in ice during strength tests.

Fig. 4. Stress and strain history of columnar-grained ice of average grain diameter of 4.5 mm at -10°C for a nominal strain rate of $1.3 \times 10^{-4} \text{ s}^{-1}$.

Fig. 5. Stress and strain history of columnar-grained ice of average grain diameter of 4.5 mm at -10°C subjected to nominal strain rate of $5 \times 10^{-6} \text{ s}^{-1}$.

t_f is the corresponding time. It is essential to recognize that the premature failures show a large scatter of results, primarily reflecting the conditions of the specimen end surfaces. Results showing stresses at 7 MN m^{-2} or higher were obtained with specimens whose end surfaces were finished extremely carefully. There is, however, significantly less scatter in the results associated with upper yield type failures. The functional dependence of σ_f on $\dot{\epsilon}_{af}$ for test results exhibiting upper yield type of failure have been presented (Sinha, 1981) as

$$\dot{\epsilon}_{af} = 1.4 \times 10^{-7} (\sigma_f)^{3.05} \quad (16)$$

Fig. 6. Dependence of stress at upper yield or premature failure by fracture on average strain rate at -10°C for columnar-grained S-2 ice of average grain diameter of 4 to 5 mm subjected to a wide range of constant cross-head displacement rates.

Fig. 7. Dependence of strain at yield or fracture on average strain rate for the results shown in Fig. 6.

Equation (16) is also shown in Fig. 6. This relation rather than the conventional σ_f versus nominal strain rate $\dot{\epsilon}_n$ has since been shown (Sinha, 1982a) to represent the dependence of upper yield stress on truly constant strain rate. The dependence of ϵ_f on $\dot{\epsilon}_{af}$ was

$$\epsilon_f = 2.3 \times 10^{-2} (\dot{\epsilon}_{af})^{0.25} \quad (17)$$

(Sinha, 1981), but it was not graphically presented and compared with the experimental data. This is now shown in Fig. 7, along with theoretical calculations for each test in Fig. 6. Experimental and corresponding theoretical results are shown as pairs

(joined by a line). The theoretical results for appropriate grain diameter of 4.5 mm are based on recorded stress histories and equations (eqns. 11a, b and c) and on the material constants in Table 1. Note the increasing deviation of the theoretically predicted strain from the measured value as the rate of loading increased. It increased not only the stress level at upper yield but also the number of microcracks in the specimens at upper yield. This corroborates, as mentioned earlier, the physical restrictions imposed on the theory. On the other hand, there is good agreement between theory and experiment, in spite of the associated high stresses, for failures for which the splitting type of fracture occurred early (indicated here as lower strains) in the test period. It was observed (Sinha, 1981) that the specimens suffered significantly less damage due to microcracking during these tests before fracture occurred. Figure 8 illustrates that the theory predicts reasonably well the experimentally observed upper yield strain for stress up to about 5 MN m^{-2} . This range of applicability extends to a stress level of about 7 MN m^{-2} for premature splitting type of failure.

The applicability of the theory to test results at other temperatures can be made by introducing the effect of temperature on $\dot{\epsilon}_v$ and α_T using eqn. (3) to (5) provided the temperature is well below the melting point. It is not certain, however, what the upper temperature limit is for the shift function given by eqn. (5). Although this subject will not be discussed in detail, it is thought that a set of examples will help to establish the merit of the theory. Figure 9 shows the stress histories obtained for columnar-grained ice at three different temperatures, using the experimental set-up described by Sinha (1981). In all these tests the cross-head rate, or nominal strain rate, used was the same, and tests were terminated soon after the upper yield stresses were reached. This was done in order to establish the internal damage in the specimens. Deformation data for these tests are presented in Fig. 10 along with the corresponding theoretical predictions based on actual stress paths and material constants in Table 1. The close agreement in each case between experiment and theory and the marked deviations from the nominal strain path are worth noticing. All the ice used for making these specimens was from different batches.

Fig. 8. Comparison between the predicted and the experimental strain at peak stress as a function of this maximum stress.

Fig. 9. Stress histories at three temperatures recorded for columnar-grained S-2 ice of average grain diameter of 4 to 5 mm subjected to a constant cross-head displacement rate of $7.5 \times 10^{-4} \text{ mm s}^{-1}$ or a nominal strain rate of $3 \times 10^{-6} \text{ s}^{-1}$.

Fig. 10. Comparison between the experimental and theoretical strain histories for the tests shown in Fig. 9. The arrow indicates the occurrence of the peak stress.

The effect of the interaction between sample and test machine is also evident in the illustrations. In each case the specimen strain rate increased monotonically with time and approached a value comparable to that of the nominal rate as the peak stress was approached. These observations are consistent with previous ones discussed in detail by Sinha (1981). As the specimen at -5°C was relatively softer than that at -10°C , the machine could impose the specified strain rate sooner at -5°C than at -10°C . On the other hand, the machine was relatively softer at -30°C in relation to the sample than it was at -10°C so that it took a longer time to approach peak strain rate. In Fig. 10 the strain paths described by $\dot{\epsilon}_{af}$ are, however, comparable to each other.

CONCLUSIONS

Simple methods of calculating elastic, delayed-elastic and permanent strain components during the most commonly applied engineering tests of uniaxial variable stress have been developed for a special class of non-linear, viscoelastic material that exhibits linear, delayed-elastic dependence on stress. The principles of computation are simple and physically comprehensible, and the method of obtaining the total strain is devoid of mathematical abstractions. The agreement of theoretical predictions for ice with experimental observations may be considered good in view of the fact that the material sometimes exhibited severe cracking activity during the tests and the effect of microcracking has not yet been introduced into the basic constitutive equation. It has been shown that the previously proposed creep equation, which was developed primarily by means of experimental data for stresses in the range of 0.5 MN m^{-2} , can be extrapolated reasonably well to very high stresses provided that the time involved is

short. Finally, the analysis corroborates the observation that constant cross-head rate does not necessarily give a constant specimen strain rate.

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