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ABSORPTION OF DIFFUSE SOUND BY A STRIP OR RECTANGULAR PATCH OF ABSORPTIVE MATERIAL

by

T. D. Northwood

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ABSORPTION OF DIFFUSE SOUND BY A STRIP OR RECTANGULAR PATCH OF ABSORPTIVE MATERIAL

by

T. D. Northwood

In a recent paper, some calculations were presented of the effective absorption coefficient of a small area of sound absorbing material set in an otherwise reflecting surface. The results have bearing particularly on the standard reverberation-chamber method for measuring the sound absorption coefficients of materials. The published results were primarily for illustration, in a discussion of reverberation chamber measurements; this note, prepared for potential users of the calculations, presents the complete set in a more usable form than was possible in publication.

The theoretical basis for the calculations reported here is outlined in an earlier paper (copy attached as Appendix A). A discussion of the final results, particularly as they affect reverberation chamber tests is given in the second paper (copy attached as Appendix B). The main body of this report will consist of a few notes on the computations and tables and graphs of the results.

NOTES ON COMPUTATION PROCEDURES

The detailed program was of course conditioned by the available computer facilities. In this instance a Bendix G-15 computer was used and the programming was done with the aid of the interpretive routine Intercom 1000. As the machine memory was not capable of taking the whole program at once, the calculations were done in three stages, employing an auxiliary magnetic tape storage for intermediate results.

The calculation to be performed (see Appendix B) was the numerical integration of

$$\bar{\alpha} = \frac{16g}{\pi} \int_0^{\pi/2} \int_0^{\pi/2} \frac{\sin^3 \phi d\phi d\theta}{(g^2 + b^2)(A^2 + B^2) + 2 \sin \phi (gA + bB) + \sin^2 \phi}, \quad (1)$$

for an adequate range of values of conductance ratio g , susceptance ratio b , and ka , which is a measure of the size of the absorbing material ($k = 2\pi/\lambda$ and $a = c'd/(c + d)$, where c and d are dimensions of a rectangular patch). Analytically derived formulae are available for $ka = 0$ and $ka = \infty$; it is desirable to make calculations for enough values of ka to permit reliable estimates throughout the intermediate region.

It is a necessary preliminary to calculate the quantities A and B for each combination of θ , ϕ , and ka , using the equations:

$$A = \int_0^{ka \sin \phi} \cos(x \sin \theta) J_0(x) dx - \frac{1}{ka \sin \phi} \int_0^{ka \sin \phi} x \cos(x \sin \theta) J_0(x) dx, \quad (2)$$

$$B = \int_0^{ka \sin \phi} \cos(x \sin \theta) N_0(x) dx - \frac{1}{ka \sin \phi} \int_0^{ka \sin \phi} x \cos(x \sin \theta) N_0(x) dx, \quad (3)^*$$

The main integration (Equation 1) was a straightforward matter. A double integration was done, employing Simpson's one-third rule for the two variables θ and ϕ . Intervals of $\pi/12$ were used for both variables. This integration was readily carried out for a good range of values of g and b .

The major task was the calculation of A and B (Equations 2 and 3) which involve θ , ϕ and ka in a complicated relation. The use of constant intervals of θ and ϕ for the final integration necessitated a different set of integration intervals for each value of $ka \sin \phi$. Hence, a complete integration was done for each combination of ka , θ and ϕ . Thus for each value of ka there were 42 calculations (plus 6 zero values for $\phi = 0$).

Though the integral B is finite throughout the interval the integrand term $N_0(x)$ becomes infinite at $x = 0$. This difficulty was disposed of by using an analytical solution for the integration in the interval $0 < x < 0.02$; in this range $\cos(x \sin \theta)$ can be taken to be unity and the integral is greatly simplified. Thus, for example, (G.N. Watson, *Treatise on the Theory of Bessel Functions*, 2nd Edition, Cambridge University Press, 1958, p. 752)

* Note: The function N_0 is the Bessel function of the second kind, designated Y_0 by some writers (e.g. Watson, Jeffreys)

$$\int_0^{0.02} \cos(x \sin \theta) N_0(x) dx \doteq \int_0^{0.02} N_0(x) dx \doteq -.06400$$

$$\text{and } \int_0^{0.02} x N_0(x) dx = \left[x N_1(x) \right]_0^{0.02} = -.000576$$

As a matter of convenience the same procedure was used also for $J_0(x)$ and $x J_0(x)$.

To investigate the variation with ka , ten values of this parameter, ranging from 2 to 50 were selected. Because of the oscillatory nature of the integrand terms, fairly small increments had to be used throughout, with the result that the calculations for large ka were rather lengthy.

The three stages of the calculation were as follows:

- Stage I. Calculate and store on magnetic tape all the required values of $J_0(x)$, $x J_0(x)$, $N_0(x)$ and $x N_0(x)$.
- Stage II. Calculate and store on magnetic tape all required values of A , B and $(A^2 + B^2)$.
- Stage III. Calculate $\bar{\alpha}$ for $g = 0.1$ to 1.0 , and $b = -0.4$ to $+0.5$, for ten values of ka from 2 to 50.

RESULTS

All the calculated values of $\bar{\alpha}$, including those analytically derived for $ka = 0$ and $ka = \infty$, are given in Tables I, II and III.

The data are also presented in perhaps more useful form in Figures 1 to 14. Extra curves have been derived by interpolation from the calculated data, to facilitate easy use of the graphs. The standard reverberation room test (ASTM C423-61T) employs a rectangular sample area, 9 ft by 8 ft, and six standard frequencies, 125, 250, 550, 1000, 2000, and 4000 cycles/sec. Values of ka for these conditions are given in Table IV. For the most part it is possible to use one of the rounded values of ka without significant error, but two extra curve sheets (for $ka = 2.95$ and $ka = 94.4$) have been derived to facilitate matters (see Table IV).

USE OF THE CURVES

The calculations give the effective absorption coefficient of any size of rectangular patch of locally-reacting material, set in an otherwise perfectly reflecting plane and exposed to a diffuse sound field. The user must first consider how closely these conditions are met. Hard plaster, concrete or masonry surfaces provide a good approximation to perfect reflection; many dense porous materials or perforated or fissured tiles mounted directly against a wall are approximately locally-reacting; if the absorbing surface is set out from the reflecting surface rather than flush with it, an error might be expected if the displacement is greater than about a quarter-wavelength. The most difficult condition is the diffuse sound field. Even in reverberation rooms, in which a high degree of diffusion is a major objective, there is a wide scatter in results (see Appendix B) which can only be attributed to diffusion problems.

Subject to the above limitations, a calculation can be made if the complex admittance ratio is known for the material. This can be derived from the complex impedance ratio using equations 4 and 5:

If the acoustic impedance ratio is denoted by $z = r + jx$ and the admittance ratio is $y = g - ib$,

$$\text{then } g = \frac{R}{R^2 + X^2}, \quad (4)$$

$$b = \frac{-X}{R^2 + X^2} \quad * \quad (5)$$

Alternatively, it can be obtained directly from impedance tube measurements. The minimum/maximum ratio and the distances from sample face to first and second minima are determined in the usual way (see ASTM C-384). Then Figure 15 may be used to obtain the conductance and susceptance ratios.

If the required value of ka falls between two of the given quantities it is usually sufficient to determine $\bar{\alpha}$'s for adjacent ka values and interpolate between them. For values of ka greater than 50 a similar procedure may be used, but using the reciprocals of ka (e.g. interpolating between $1/ka = 0.02$ and $1/ka = 0$).

* The discrepancy in sign arises from the different origins of the impedance and admittance formulae.

TABLE I

EFFECTIVE ABSORPTION COEFFICIENTS FOR NEGATIVE VALUES OF SUSCEPTANCE RATIO, b

ka	b = - 0.1				b = - 0.2				b = - 0.4			
	g=0.1	g=0.2	g=0.5	g=1.0	g=0.1	g=0.2	g=0.5	g=1.0	g=0.1	g=0.2	g=0.5	g=1.0
0	0.800	1.600	4.000	8.000	0.800	1.600	4.000	8.000	0.800	1.600	4.000	8.000
2	.602	1.048	1.769	2.037	.534	0.942	1.647	1.955	.420	0.761	1.413	1.782
4	.567	0.944	1.454	1.546	.500	.850	1.363	1.493	.386	.680	1.176	1.374
6	.548	.892	1.310	1.335	.483	.804	1.233	1.295	.370	.641	1.068	1.198
8	.537	.860	1.225	1.224	.473	.777	1.160	1.191	.361	.620	1.002	1.108
12	.523	.822	1.134	1.106	.461	.746	1.080	1.080	.352	.599	0.944	1.011
16	.515	.801	1.087	1.050	.456	.730	1.039	1.028	.349	.588	.919	0.968
20	.510	.788	1.059	1.020	.453	.722	1.015	1.001	.348	.582	.902	.945
26	.503	.774	1.033	0.992	.448	.710	0.993	0.974	.346	.576	.886	.923
32	.498	.762	1.013	.942	.443	.699	.974	.957	.344	.571	.873	.910
50	.486	.740	0.976	.939	.436	.682	.944	.930	.342	.564	.852	.888
∞	.471	.705	.931	.905	.431	.666	.909	.896	.340	.558	.832	.860

TABLE II
EFFECTIVE ABSORPTION COEFFICIENTS FOR
ZERO SUSCEPTANCE RATIO, b

ka	g							
	0.10	0.15	0.20	0.30	0.50	0.70	0.80	1.0
0	0.800	1.200	1.600	2.400	4.000	5.600	6.400	8.000
2	.680	0.942	1.162	1.500	1.891	2.061	2.098	2.114
4	.639	.862	1.039	1.290	1.539	1.613	1.618	1.591
6	.616	.820	0.977	1.189	1.378	1.416	1.408	1.368
8	.602	.794	.939	1.128	1.282	1.306	1.294	1.250
12	.583	.760	.890	1.054	1.178	1.185	1.170	1.124
16	.572	.740	.862	1.013	1.122	1.123	1.108	1.065
20	.564	.727	.843	0.987	1.090	1.092	1.075	1.032
26	.555	.713	.823	.961	1.058	1.061	1.044	1.001
32	.547	.699	.807	.939	1.033	1.036	1.022	0.980
50	.530	.674	.775	.899	0.993	0.996	0.983	.950
∞	.489	.623	.720	.842	.939	.949	.941	.910

TABLE III

EFFECTIVE ABSORPTION COEFFICIENTS FOR POSITIVE VALUES OF SUSCEPTANCE RATIO, b

ka	b = + 0.1				b = + 0.2				b = + 0.3			
	g=0.1	g=0.2	g=0.5	g=1.0	g=0.1	g=0.2	g=0.5	g=1.0	g=0.1	g=0.2	g=0.5	g=1.0
0	0.800	1.600	4.000	8.000	0.800	1.600	4.000	8.000	0.800	1.600	4.000	8.000
2	.764	1.281	2.008	2.184	.852	1.399	2.116	2.244	.937	1.506	2.206	2.293
4	.709	1.128	1.611	1.629	.769	1.199	1.666	1.657	.804	1.242	1.700	1.673
6	.679	1.051	1.431	1.393	.723	1.102	1.466	1.408	.738	1.120	1.478	1.413
8	.658	1.001	1.326	1.268	.691	1.038	1.348	1.276	.693	1.041	1.348	1.275
12	.631	0.940	1.206	1.134	.651	0.958	1.214	1.133	.636	0.941	1.200	1.129
16	.612	.901	1.144	1.072	.622	.907	1.144	1.070	.594	.877	1.172	1.059
20	.599	.875	1.104	1.038	.600	.872	1.098	1.034	.564	.834	1.072	1.021
26	.586	.850	1.065	1.005	.578	.839	1.055	0.998	.533	.793	1.026	0.985
32	.572	.827	1.039	0.983	.559	.810	1.025	.976	.508	.760	0.995	.962
50	.545	.783	0.995	.951	.517	.755	0.976	.942	.459	.699	.941	.927
∞	.471	.705	.931	.905	.431	.666	.909	.896	.388	.614	.874	.881

ka	b = + 0.4				b = + 0.5			
	g=0.1	g=0.2	g=0.5	g=1.0	g=0.1	g=0.2	g=0.5	g=1.0
0	0.800	1.600	4.000	8.000	0.800	1.600	4.000	8.000
2	1.005	1.589	2.274	2.329	1.043	1.637	2.313	2.351
4	0.808	1.251	1.710	1.679	0.784	1.226	1.695	1.672
6	.723	1.105	1.468	1.408	.683	1.058	1.435	1.392
8	.665	1.009	1.326	1.264	.614	0.950	1.284	1.244
12	.590	0.891	1.166	1.113	.526	.817	1.116	1.088
16	.538	.816	1.081	1.040	.469	.738	1.028	1.016
20	.502	.769	1.029	1.000	.433	.689	0.974	0.976
26	.468	.723	0.981	0.964	.401	.641	.927	.940
32	.442	.689	.950	.941	.377	.610	.895	.916
50	.396	.633	.898	.905	.340	.562	.844	.881
∞	.340	.558	.832	.860	.298	.502	.781	.834

TABLE IV

LIST OF DATA FOR CALCULATING EFFECTIVE ABSORPTION OF
STANDARD SAMPLE AT STANDARD FREQUENCIES
(PER ASTM C423 - 61 T)

Frequency Cycles/sec	ka Values		Figure
	Exact	Nearest Available	
125	2.95	2.95	2
250	5.9	6	4
500	11.8	12	6
1000	23.6	26	9
2000	47.2	50	11
4000	94.4	94.4	12

FIGURE 2 $ka = 2.95$ (STANDARD SAMPLE AREA AT 125 c/s)

FIGURE 3 $ka = 4$

FIGURE 4 $ka = 6$ (STANDARD SAMPLE AREA AT 250 c/s)

FIGURE 5 $ka = 8$

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FIGURE 6 $k_a = 12$ (STD SAMPLE AREA AT 500 c/s)

FIGURE 7 $ka = 16$

FIGURE 8 $ka = 20$

FIGURE 9 $k_a = 26$ (STD SAMPLE AREA AT 1000 c/s)

FIGURE 10 $ka = 32$

FIGURE 11 $k_a = 50$ (STD SAMPLE AREA AT 2000 c/s)

FIGURE 12 $ka = 94.4$ (STD SAMPLE AREA AT 4000 c/s)

FIGURE 13 k_a INFINITE

FIGURE 14 SUSCEPTANCE RATIO $b=0$, VARIOUS ka

FIGURE 15
CHART FOR DETERMINING CONDUCTANCE AND SUSCEPTANCE RATIOS FROM IMPEDANCE TUBE MEASUREMENTS

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APPENDIX A

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Absorption of Sound by a Strip of Absorptive Material in a Diffuse Sound Field*

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(Received February 5, 1959)

The random incidence sound absorption coefficient is calculated for a narrow strip of absorbing material set in an otherwise reflecting plane. The assumptions made are that the material is of the locally reacting type, with a real normal admittance. The effect of an imaginary component of admittance is discussed qualitatively.

The results show the increase in absorption coefficient that occurs at small widths due to diffraction. Consideration is also given to the absorption of rectangular patches having both dimensions in the diffraction range. The standard reverberation room measurement of sound absorption is examined.

INTRODUCTION

THE performance of a sound absorbing material is generally determined by placing a sample in a reverberation room and observing its effect on the reverberation time in the room. The sound absorption coefficients for the material are then calculated from the two sets of reverberation times using the Sabine or Norris-Eyring reverberation formula. The coefficients thus obtained may be used with reasonable success in the many acoustical problems for which the reverberation theory is valid. There is, however, one notable weakness in the procedure. The sample that can conveniently be tested in existing reverberation rooms is a relatively small patch the dimensions of which are a few wavelengths or less, depending on frequency. In this region diffraction effects may be expected to occur, resulting in measured absorption coefficients that depend on the dimensions of the sample. It is desirable to know the extent of this effect, and to be able to correct for it in typical applications.

Most of the laboratories that operate reverberation rooms have made experimental studies of the effect of sample size. Of the published material¹⁻³ the most

comprehensive is Chrisler's well-known paper. There are difficulties, however, in studying an adequate range of sample sizes within the confines of established reverberation room techniques. In particular, it is difficult to approach the limiting value corresponding to an infinite area. To achieve the diffuse field which is basic to the reverberation theory, the total absorption in the room must not become too high; and to avoid effects peculiar to the room boundaries the sample must not approach closer than about half a wavelength to an edge or corner of the room.⁴ For very small samples, the only difficulty is in producing a measurable change in the absorption in the room. One may avoid this difficulty by using several small samples, but there will probably be an interaction unless they are spaced several wavelengths apart.

One of the earliest theoretical studies of the problem was by Pellam,⁵ who developed an expression for the absorption of an infinitely long narrow strip for plane waves incident in a direction perpendicular to the axis of the strip. Levitas and Lax⁶ also investigated this problem, and set up a variational method applicable to more complicated cases, one of which will be dealt with in this paper. Both of these studies apply to a material of the "locally reacting" type the behavior of which can be specified by its specific normal impedance or admittance.

* This paper is a contribution of the Division of Building Research, National Research Council, Canada, and is published with the approval of the Director of the Division.

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¹ V. L. Chrisler, J. Research Natl. Bur. Standards 13, 169 (1934).

² Paul Sabine, J. Acoust. Soc. Am. 6, 239 (1935).

³ Norris, Nixon, and Parkinson, J. Acoust. Soc. Am. 9, 234 (1938).

⁴ R. V. Waterhouse, J. Acoust. Soc. Am. 29, 544-547 (1956).

⁵ J. R. Pellam, J. Acoust. Soc. Am. 11, 396 (1940).

⁶ A. Levitas and M. Lax, J. Acoust. Soc. Am. 23, 316-322 (1951).

FIG. 1. Geometrical formulation of the problem. The strip lies in the XY plane, its center line coinciding with the Y axis. OP is the propagation vector k for a typical incident plane wave, with components k_x , k_y , and k_z . OA is the quantity referred to in the text as $\kappa = (k^2 - k_y^2)^{1/2} = k \sin \phi$.

Cook⁷ has considered both the strip and the circular patch, and has worked out the latter for both normal and random incidence. But this was achieved at the expense of assuming a piston absorber, the whole surface of which must act in phase, and except for normally incident sound the absorption coefficient must approach zero for large dimensions. Hence the results for random incidence do not throw much light on the practical case. More recently Cook⁸ has been able to generalize the method by replacing the piston absorber with one in which the surface motion may be specified by an appropriate Fourier expansion. The components may be selected to fit the boundary condition of a specified normal admittance. This work, when completed, will represent a complete solution to the problem, exact in form although it will involve the evaluation of series solutions.

The present work is a direct extension of the method of Levitas and Lax to determine the random incidence absorption of a strip. The procedure will be developed very briefly with frequent reference to their paper. They consider an absorbing strip of infinite length set in an otherwise perfectly reflecting plane, the XY plane (Fig. 1). The center line of the strip coincides with the Y axis, and its width is a . Its absorptive properties are specified by a normal admittance ratio η (eventually assumed to be real). The first step in the solution is to use Green's-function techniques to express the pressure at any point as an integral involving the pressure on the surface of the strip. This leads to an integral expression for the pressure on the strip itself. No attempt is made to solve this, but it is used in a variational procedure leading to expressions for the sum and the ratio of scattering and absorption cross sections. These are combined to yield explicit expressions for scattering and absorption cross sections, in terms of wavelength, width of strip, and its normal

admittance ratio. The solution is developed for propagation in the XZ plane, but it is shown that incidence out of the XZ plane may be dealt with by considering only the component normal to the Y axis (since the absorption of the material is assumed to depend only on the normal impedance).

THEORY FOR OBLIQUE INCIDENCE

Levitas and Lax work out the complete solution for normal incidence only. The more general case is readily evaluated by retaining the angular variable in their Eq. (36) and thereafter. § Incidence in the XZ plane is first considered. Equations (38) and (49) retain the same form except that the integrals A and B are modified by a $\cos(x \sin \theta)$ factor relating to the normal impedance assumption [see Eq. (100) below].

Thus

$$\sigma_s/\sigma_a = |\eta|^2 A / \text{Re}(\eta) \quad (38)$$

and

$$\sigma_t = \sigma_s + \sigma_a = 4a \text{Re} \left(\frac{\eta}{1 + \eta(A + iB)} \right), \quad (49)$$

where σ_s , σ_a , and σ_t are, respectively, the scattering, absorption, and total cross sections (per unit length).

FIG. 2. Absorption coefficient of infinite strip of width a and real admittance ratio δ .

⁷ R. K. Cook, J. Acoust. Soc. Am. 29, 324-329 (1957).

⁸ R. K. Cook, J. Acoust. Soc. Am. 30, 697 (A) (1958).

§ Equations numbered below (100) are from Levitas and Lax (reference 6).

Combining these, putting $\eta = \delta + i\xi$,*

$$\sigma_a(\theta) = \frac{4a\delta}{1 + (\delta^2 + \xi^2)(A^2 + B^2) + 2(\delta A - \xi B)}$$

Following Levitas and Lax, this expression is modified to take account of incidence out of the XZ plane by replacing η with an effective admittance ratio

$$\eta' = \eta k / \kappa = \eta / \sin \phi.$$

This amounts to considering only the component of the incident wave parallel to the XZ plane for calculating both incident and absorbed energy. To obtain the absorption cross section, defined as the energy absorbed from an incident wave of unit intensity, the expression must be multiplied by $\sin \phi$ to restore the variation of incident energy with angle ϕ . The limits of the integrals A and B are also affected since $d = \frac{1}{2} \kappa a = \frac{1}{2} k a \sin \phi$. With these changes, the absorption cross section is expressed as a function of θ and ϕ

$$\sigma_a(\theta, \phi) = \frac{4a\delta \sin^2 \phi}{(\delta^2 + \xi^2)(A^2 + B^2) + 2 \sin \phi (\delta A - \xi B) + \sin^2 \phi}, \quad (100)$$

where

$$A = \int_0^{ka \sin \phi} \cos(x \sin \theta) J_0(x) dx - \frac{1}{ka \sin \phi} \int_0^{ka \sin \phi} x \cos(x \sin \theta) J_0(x) dx$$

FIG. 3. Infinite-area absorption coefficient as a function of δ and ξ (real and imaginary parts of complex admittance ratio).

* Replaced by $y = g - ib$ in Appendix B and main report.

and

$$B = \int_0^{ka \sin \phi} \cos(x \sin \theta) N_0(x) dx - \frac{1}{ka \sin \phi} \int_0^{ka \sin \phi} x \cos(x \sin \theta) N_0(x) dx.$$

This is the energy absorbed by a unit length of strip for an incident plane wave of unit intensity.

Random Incidence Absorption Coefficient

Consider now an incident sound field consisting of a uniform distribution of such incident waves (of unit intensity) for all angles in the hemisphere. The incident energy density on the strip for one such wave will be $\cos \theta \sin \phi$. The energy per unit area absorbed from such a wave will be $\sigma_a(\theta, \phi)/a$. The random incidence absorption coefficient is obtained by integrating each of these quantities over the hemisphere and forming the quotient (total energy absorbed per unit area)/(total incident energy density). The denominator is just $\pi/4$, so that

$$\bar{\alpha} = \frac{16\delta}{\pi} \int_0^{\pi/2} \int_0^{\pi/2} \frac{\sin^2 \phi \cdot \sin \phi d\phi d\theta}{(\delta^2 + \xi^2)(A^2 + B^2) + 2 \sin \phi (\delta A - \xi B) + \sin^2 \phi}. \quad (101)$$

When a approaches zero, A and B also approach zero and Eq. (101) becomes $\bar{\alpha}_0 = 8\delta$. This is the value usually obtained for a strip or patch of absorbing material small enough that it does not significantly alter the sound field at the otherwise reflecting surface.

When the strip width becomes very large $B \rightarrow 0$ and $A \rightarrow (1 - \sin^2 \theta)^{-1/2} = 1/\cos \theta$. Equation (101) then reduces to the more symmetric form

$$\bar{\alpha}_\infty = \frac{16\delta}{\pi} \int_0^{\pi/2} \int_0^{\pi/2} \frac{\cos^2 \theta \sin^2 \phi \cdot \sin \phi d\phi d\theta}{(\delta^2 + \xi^2) + 2\delta \cos \theta \sin \phi + \cos^2 \theta \sin^2 \phi}. \quad (102)$$

Noting that the term $\cos \theta \sin \phi = \cos I$, where I is the angle of incidence, the integral is readily evaluated by rotating the coordinates so that the polar axis is normal to the surface. The result is

$$\bar{\alpha}_\infty = 8\delta \left\{ 1 - \delta \ln \left(\frac{1 + 2\delta + \delta^2 + \xi^2}{\delta^2 + \xi^2} \right) + \frac{\delta^2 - \xi^2}{\xi} \tan^{-1} \left(\frac{\xi}{\xi^2 + \delta^2 + \delta} \right) \right\}. \quad (103)$$

This agrees with Morse's expression⁹ for the absorption

⁹ P. M. Morse, *Vibration and Sound* (McGraw-Hill Book Company, Inc., New York, 1948), second edition, p. 388.

FIG. 4. Experimental measurements of $\bar{\alpha}$ for long strips and approximately square patches of same material. (Samples described in Table I. Curves are theoretical results.)

coefficient of an infinite area. For real admittance this reduces to

$$\bar{\alpha}_\infty' = 8\delta \left\{ 1 - 2\delta \ln \left(\frac{1+\delta}{\delta} \right) + \frac{\delta}{1+\delta} \right\}. \quad (104)$$

Equation (101) has been evaluated for a range of values of ka and for real admittance, with the results shown in Fig. 2. Values for infinite strip width, calculated from Eq. (104), are plotted on the right-hand ordinate.

It will be seen that the absorption coefficient increases continuously as the width decreases, without the periodic fluctuations of the normal-incidence case. This is an expected consequence of including obliquely incident waves in the calculation. Another result of the oblique incidence is the fact that the effect is significant even when the strip is several wavelengths wide.

Some information on the effect of an imaginary component of admittance may be obtained by considering the formulas for very narrow and very wide strips. For very narrow strips the absorption coefficient depends only on the real part. For infinite width the absorption is given by Eq. (103), from which the curves of Fig. 3 are plotted, and it will be seen that the coefficient is decreased by the presence of an imaginary component. Thus, for a complex admittance the curves will start as in Fig. 2, depending on the real part of

the admittance, and for large widths will droop lower than in Fig. 2 by an amount depending on the magnitude of the imaginary part. Hence the curves of Fig. 2, for real admittances, indicate the minimum variation of absorption with strip width that can occur.

Absorption of a Rectangular Patch

The results for an infinite strip presumably can be applied directly to finite strips when the length is greater than a few wavelengths. It is important to know the variation that occurs when *both* dimensions of a rectangular patch are small enough to produce diffraction effects. An empirical way of adapting the infinite strip results to this case is to replace the strip width by a parameter that depends equally on the two dimensions of the rectangle and which reverts to the strip formula when either dimension becomes very large. The simplest procedure is to replace a by $bc/(b+c)$, where b and c are the dimensions of the rectangle. This gives the correct limiting values and will not be greatly in error when the patch takes the form of a long but finite strip. The most serious deviation is likely to be for a square patch, which is given the same diffraction effect as an infinite strip of width equal to half the side of the square. Experimental studies with squares and long strips indicate that this empirical formula is a good approximation (Fig. 4).

FIG. 5. Experimental measurements of $\bar{\alpha}$ for rectangular patches of dimensions $b \times c$. (Samples described in Table I. Curves are theoretical results. Points in parentheses are calculated.)

EXPERIMENTAL EVIDENCE

Over the years considerable experimental information has been accumulated by the senior author on the variation of absorption with area. For three materials impedance tube measurements were also available, and since the materials were mounted in the same way for both measurements the results should be consistent. Experimental results for these materials are plotted in Figs. 4 and 5, with the theoretical curves of Fig. 2 for comparison.

In Fig. 4 three sets of results for long strips and nearly square patches of the same material are shown, demonstrating that the empirical formula for rectangular patches is reasonably independent of sample shape. Additional results are plotted in Fig. 5.

Table I gives impedance tube and other information for the three samples. Values of $\bar{\alpha}_0$ and $\bar{\alpha}_\infty$, corresponding to very small and very large areas, are given in the table and are also plotted in Figs. 4 and 5, where they may be compared with the experimental points. Comparison of $\bar{\alpha}_\infty$ and $\bar{\alpha}_\infty'$ (obtained by neglecting ξ)

TABLE I. Measurements and calculations for experimental materials.

Sample	Material	Freq. cps	δ	Tube data*					Room data Approx. δ
				ξ	$\bar{\alpha}_0$	$\bar{\alpha}_\infty$	$\bar{\alpha}_\infty'$		
1	Glass fiber 2 in. thick	500	0.31	-0.47	2.5	0.70	0.85	0.4	
		1000	0.84	-0.39	6.7	0.83	0.94	0.8	
2	Perforated wood fiber 1½ in. thick	500	0.37	-0.22	2.56	0.86	0.89	-0.35	
		1000	0.23	+0.13	1.84	0.75	0.77	0.17	
3	Perforated wood fiber ¾ in. thick	500	0.09	-0.03	0.71	0.47	0.48	0.12	
		1000	0.14	-0.03	1.12	0.60	0.60	0.18	

* Normal admittance ratio $\eta = \delta + i\xi$.

indicates the deviation from the real admittance curves that may be expected for large areas. The last column is obtained by comparing the experimental results with the theoretical curves and estimating the admittance ratios for the samples.

More experimental work is planned, using more carefully designed samples. In the meantime the agreement is good enough to suggest that for certain types of material, the random incidence absorption can be predicted from impedance tube information. The practical significance of this is limited by the fact that the absorption of many materials depends on large-scale effects such as mounting and backspace. One must also rule out materials for which the normal impedance

FIG. 6. Infinite-area absorption vs absorption of standard sample (9 ft by 8 ft).

assumption does not apply. Nevertheless the fact that there appears to be correspondence where there should be helps to close the gap that has existed between impedance tube measurements and reverberation room results.

AREA EFFECT AND REVERBERATION ROOM MEASUREMENTS

For sound absorption tests most reverberation rooms on this continent use a standard sample having the dimensions 9 ft by 8 ft. Applying the foregoing results to a patch of these dimensions gives the curves of Fig. 6, which relate standard-area coefficients and infinite-area coefficients. The effect is a substantial one, which should not be ignored. There are two possible corrective procedures. One can correct standard-area measurements to the infinite-area values, or one can continue to quote standard-area measurements in the expectation that the user will calculate the absorption of each given area. Possibly both values might be given, the infinite-area one for routine calculations and the standard-area value to be used as a starting point when more precise calculations are required.

R. K. Cook has suggested that the term "absorption coefficient," implying a material property that is independent of area, might be replaced by the "absorption cross section," which is simply the total absorption of a given patch of material. For routine applications involving large areas of treatment perhaps this would be an unduly complicated procedure. But for patches of less than a few wavelengths in dimension, or for special configurations such as space absorbers, the absorption cross section is a useful quantity.

APPENDIX B

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Absorption of Diffuse Sound by a Strip or Rectangular Patch of Absorptive Material*

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Previous calculations of the effective absorption coefficient of a rectangular patch of absorptive material have been extended in several respects. The new results are applicable to locally reacting materials for any size of patch and for a wide range of complex admittances. Specific use of the results is made in an examination of the reverberation-room method of measuring sound-absorption coefficients.

A PREVIOUS paper¹ presented calculations of the absorption of diffuse sound by a narrow strip of absorptive material set in an otherwise reflecting surface. Although the formulas used were developed in the first instance for an infinite strip, it was found experimentally that they could be modified to apply to a rectangular patch. The results have been used in recent reverberation-room studies with sufficient success to warrant more-complete calculations. The expanded results are reported here.

The principal extension of the calculation has been from real to complex admittances. In addition, following experience with the earlier results, the values of the various parameters were selected to permit more-accurate interpolations in some regions. Finally, the numerical-integration procedure was refined somewhat, so that the new results are more accurate than those reported previously.

With the extensions the complete results are rather voluminous for brief presentation. In this paper, the more important functional relations are demonstrated

and discussed, particularly with reference to the reverberation-room measurement of sound absorption. A manuscript containing full details is available from the author.

NOTATION

When the results were extended to include complex admittances, it was realized that the previous expression for the admittance ratio $\eta = \delta + i\xi$ gave the wrong sign for the susceptance ratio as compared to the conventions used by most writers. As the absorption of a small patch depends on the sign of the susceptance, the notation in this paper has been revised to agree with usual practice; the admittance ratio is now denoted by $y = g - ib$. The mathematical expressions, recapitulated here, incorporate this change and incidentally correct one or two typographical errors in the earlier work.

BASIC EQUATIONS

The absorption coefficient for diffuse sound incident on a long absorptive strip set in a reflecting plane is given by

$$\bar{\alpha} = \frac{16g}{\pi} \int_0^{\pi/2} \int_0^{\pi/2} \frac{\sin^3 \phi d\phi d\theta}{(g^2 + b^2)(A^2 + B^2) + 2 \sin \phi (gA + bB) + \sin^2 \phi}, \quad (1)$$

where

$$A = \int_0^{ka \sin \phi} \cos(x \sin \theta) J_0(x) dx - \frac{1}{ka \sin \phi} \int_0^{ka \sin \phi} x \cos(x \sin \theta) J_0(x) dx, \quad (2)$$

$$B = \int_0^{ka \sin \phi} \cos(x \sin \theta) N_0(x) dx - \frac{1}{ka \sin \phi} \int_0^{ka \sin \phi} x \cos(x \sin \theta) N_0(x) dx,$$

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¹ T. D. Northwood, M. T. Grisar, and M. A. Medcof, J. Acoust. Soc. Am. 31, 595 (1959).

a is the width of the strip, λ is the wavelength (in same units as a), and $k=2\pi/\lambda$.

Numerical integrations were done for eight values of g between 0.1 and 1.0, for eleven values of b from -0.5 to $+0.5$, and for nine values of ka between 2 and 50. These permit accurate determinations of effective absorption coefficients for any size of rectangular patch and for the range of admittances commonly encountered. For limiting values of ka , Eq. (1) reduces to simpler forms: when ka is very large, it reverts to Morse's formula for the statistical absorption coefficient of an infinite area; when ka approaches zero, the effective absorption coefficient becomes $8g$.

The results are adapted to a rectangular patch by putting $a=cd/(c+d)$, where c and d are the dimensions of the patch. This expression, which reverts to the infinite-strip form when either dimension becomes very large, is found to fit experimental data for squares and long strips.¹ It is similar also to the parameters used by Kuhl² and others for experimental studies.

RESULTS

Figure 1 shows the variation of absorption coefficient with the parameter $1/(ka)$. The reciprocal parameter is chosen in order to show the transition to infinite area ($1/(ka)=0$). Particular values of $1/(ka)$, corresponding to the standard sample size used in absorption tests and the standard test frequencies,³ are marked off along the abscissa. It will be noted that there is a

FIG. 1. Effective absorption as a function of sample dimensions for a range of admittance ratios. Solid lines are for real-admittance ratios ($b=0$); broken lines show range of susceptance ratios for one value of conductance ratio.

² W. Kuhl, *Acustica* 10, 264 (1960).

³ "Tentative Method of Practice for Sound Absorption of Acoustical Materials in Reverberation Rooms," Am. Soc. Testing Mater. ASTM C423-60T (1960).

substantial difference between the coefficients obtained for the standard sample and for an infinite area at all but the highest test frequencies. The two broken lines are for complex admittances; the principal point to observe is that, although these curves are similar in form to the real-admittance curves over most of the range, they deviate substantially in the transition to infinite area.

FIG. 2. Effective absorption coefficient of standard 9- x 8-ft sample at 250 cps.

FIG. 3. Effective absorption coefficient of standard 9- x 8-ft sample at 500 cps.

Further information concerning the imaginary component is shown in Figs. 2-5, which include most of the results for four of the standard test conditions. Whereas for an infinite area (Fig. 6) the absorption is maximum for zero susceptance, the peak shifts toward positive susceptance as the sample size, relative to wavelength, decreases.

ABSORPTION OF A RECTANGULAR PATCH OF MATERIAL

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COMPARISON WITH REVERBERATION-ROOM RESULTS

The most important application of the foregoing calculations is to provide an independent check on the reverberation-room method for measuring sound-absorption coefficients. The reverberation-room method purports to measure the random-incidence absorption

FIG. 4. Effective absorption coefficient of standard 9' X 8-ft sample at 1000 cps.

FIG. 5. Effective absorption coefficient of standard 9' X 8-ft sample at 2000 cps.

coefficient of a sample of material, but the intricacies of the method, based more on empirical attempts to obtain repeatable results than on theoretical considerations, leave some doubt regarding the quantity that finally emerges. For a special category of materials, it is now possible to calculate from impedance-tube measurements the absorption coefficients that should be obtained in the reverberation room.

FIG. 6. Absorption coefficient of infinite area of material.

The materials for which the comparison can be made must fulfill three requirements: (1) They must be of the locally reacting type; i.e., obliquely incident waves must not be propagated obliquely within the material. (2) The mounting condition must be the same in reverberation-room and impedance-tube tests, and must not introduce deviations from requirement (1). (3) The material should not be too thick, unless it can be set in a cavity with its face flush with the surrounding surface, since the diffraction calculation is based on the interaction of the absorbing surface and an adjacent reflecting surface. These requirements seem very stringent, but they do not rule out all materials. Any reasonably dense porous material is approximately locally reacting, as attenuation and refraction reduce oblique transmission in the material to negligible importance. Perforations or fissures part way through such materials will not rule them out, except at high frequencies where the perforation spacing relative to a wavelength may be important. Ruled out completely, of course, are all types of mountings involving a space behind the surface material. The thickness of the material when mounted on, rather than set into, the surrounding surface may become important at the higher frequencies where it becomes comparable to a wavelength.

It is a necessary preliminary to note that neither the current ASTM recommended practice nor the proposed ISO standard will ensure that any two laboratories will obtain the same results. Results of the recent round robin reported by Kosten⁴ illustrate this point; the spread in results obtained by eight laboratories that conform to the proposed ISO method is shown in Figs. 7-9. Reasons for this spread are currently being sought by several laboratories. In the meantime, it is of interest to compare the *average* of the round-robin

⁴ C. W. Kosten, *Acustica* 10, 400 (1960).

FIG. 7. Calculated and measured absorption coefficients for 2- $\times$ 2-m sample of Sillan in reverberation room. (See Ref. 4.)

results with the values predicted from impedance measurements and calculations for the finite sample sizes. These quantities are also shown on Figs. 7-9. Impedance-tube measurements were made by the author on several samples of the material tested in his laboratory (one of the eight in Kosten's report). The average results are given in Table I.

It appears that there is a slight discrepancy at the top frequencies, where reverberation-room results are systematically higher than calculated values. Apart from this, there is substantial agreement, and it ap-

FIG. 8. Calculated and measured absorption coefficients for 3- $\times$ 2.5-m sample of Sillan in reverberation room. (See Ref. 4.)

FIG. 9. Calculated and measured absorption coefficients for 3- $\times$ 4-m sample of Sillan in reverberation room. (See Ref. 4.)

pears that the average of results for several reverberation rooms is indeed the value that would be predicted from acoustical-impedance data. Thus, it is possible that the reverberation-room method, with suitable refinements to reduce the present scatter in results, may yield results that have a definite physical significance.

Application of Reverberation-Room Measurements

Assuming that the remaining difficulties in the reverberation-room method can be disposed of, so that all laboratories can consistently measure the random-incidence absorption of a standard size of sample, it is pertinent to consider what uses can be made of such measurements. Should the effective coefficients for a standard sample be the final quoted quantities? These data involve the minimum of processing and form an adequate basis for comparing materials; but the more common applications of materials involve areas larger than the standard sample, for which the coefficients should be smaller. Moreover, there is the awkward problem that when the reverberation method is cor-

TABLE I. Admittance and absorption calculations for Sillan samples:

Frequency	125	250	500	1000	2000	4000
Conductance ratio	0.025	0.10	0.33	0.58	0.58	0.59
Susceptance ratio	0.15	0.29	0.42	0.20	0.09	0.00
Absorption coefficients						
4 sq m	0.22	0.78	1.15	1.13	1.03	0.98
7.5 sq m	0.21	0.75	1.05	1.08	1.00	0.97
12 sq m	0.21	0.71	0.99	1.04	0.98	0.96
infinite area	0.13	0.39	0.72	0.92	0.94	0.95