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National Research Council Canada   Conseil national de recherches Canada

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Performance of concrete sidewalks: field studies

B. Rajani and C. Zhan

Abstract: This article examines the different failure modes of concrete sidewalks through an extensive field survey of sidewalk inventories in major cities in the Canadian Prairies. The major form of sidewalk damage is longitudinal cracks. There was no correlation of either the liquid limit or the plastic limit of the soil beneath the sidewalk with the type of sidewalk damage observed in each city. However, the extent of longitudinal sidewalk damage increases when the sidewalks are founded on soils with high plasticity index. Two failure modes (hogging and sagging) are closely examined through the analysis of observed vertical surface movements to explain the occurrence of longitudinal cracks. It is concluded that the rigid body movements (both uniform vertical movement and tilt) are mainly a consequence of frost penetration beneath the sidewalk. Uniform vertical movement is not very sensitive to moisture changes in the soil underneath the sidewalk. The dominant mode of deformation for sidewalks is hogging. The seasonal temperatures have considerable impact on the interaction of the sidewalk with the underlying soil, but an indirect estimate of the flexural strain or differential movement ratio is not sufficient to determine if they are large enough to exceed the tensile strain of concrete.

Key words: concrete sidewalks, longitudinal cracking, frost, failure modes.

Résumé : Cet article examine les différents modes de rupture des trottoirs de béton à l'aide d'une inspection étendue sur le terrain des inventaires de trottoirs dans les principales villes des Prairies Canadiennes. La principale forme de dommage des trottoirs est causée par les fissures longitudinales. Il n'y avait pas de corrélation entre la limite liquide ou la limite plastique du sol sous les trottoirs et le type de dommage observé dans chaque ville. Cependant, l'étendue des dommages longitudinaux des trottoirs augmente s'ils sont construits sur des sols ayant un indice de plasticité élevé. Deux modes de rupture (cintrage et affaissement) sont examinés de près à l'aide de l'analyse de mouvements de surface verticaux observés, afin d'expliquer la formation des fissures longitudinales. Il a été conclu que les mouvements de corps rigide (mouvement vertical uniforme ou incliné) sont essentiellement causés par la pénétration du gel sous le trottoir. Le mouvement vertical uniforme n'est pas sensible aux changements d'humidité dans le sol sous le trottoir. Le mode dominant de déformation des trottoirs est le cintrage. Les températures saisonnières ont un impact considérable sur l'interaction entre le trottoir et le sol. Cependant, l'estimation indirecte de la déformation en flexion ou du mouvement différentiel n'est pas suffisante pour déterminer s'ils sont suffisamment élevés pour dépasser la déformation ultime en tension du béton.

Mots clés : trottoirs de béton, fissuration longitudinale, gel, modes de rupture.
[Traduit par la Rédaction]

Introduction

It is now widely recognized that infrastructure renewal represents a major thrust of activities for regional municipalities in the 1990s. Damage to Canadian infrastructure amounts to a loss of considerable revenue. One such problem is the cracking of concrete sidewalks in some of the major Canadian cities. This problem has become more acute as regional municipalities are having to pay attention to the repair or replacement costs of sidewalks. Any significant improvement in the service life of sidewalks would reduce capital cost outlays and make budgets available for other more pressing needs.

There are approximately 100 000 km of sidewalks in Canadian cities, which have a replacement value approaching $10–12 billion. It is estimated that the cities need to replace between 15% and 30% of their sidewalk inventory. The present average service life of sidewalks is 20 years. Sidewalks in some western Canadian cities fail prematurely and some as soon as 5 years after construction. Appropriate construction practice can extend the life of sidewalks and thus decrease the annual replacement expenditures, which are in the range of $150 million per year.

In various prairie cities, major evidence of damage to the sidewalks appears predominantly in the form of longitudinal cracks along the centre of the sidewalk. On occasion there may also be transverse cracks. Cracks can extend several tens of metres within a single city block and can occur as early as 1 year after construction. A closer look at the pattern of cracks indicates that these cracks can be classified in three major categories (Fig. 1). Corner cracks are observed (Fig. 1*a*) to occur probably as a result of inadequate compaction of the underlying subgrade at the time of construction. Corner sidewalk cracks also occur in regions where longitudinal cracking is not predominant (e.g., Ottawa, Montreal). Perhaps the most
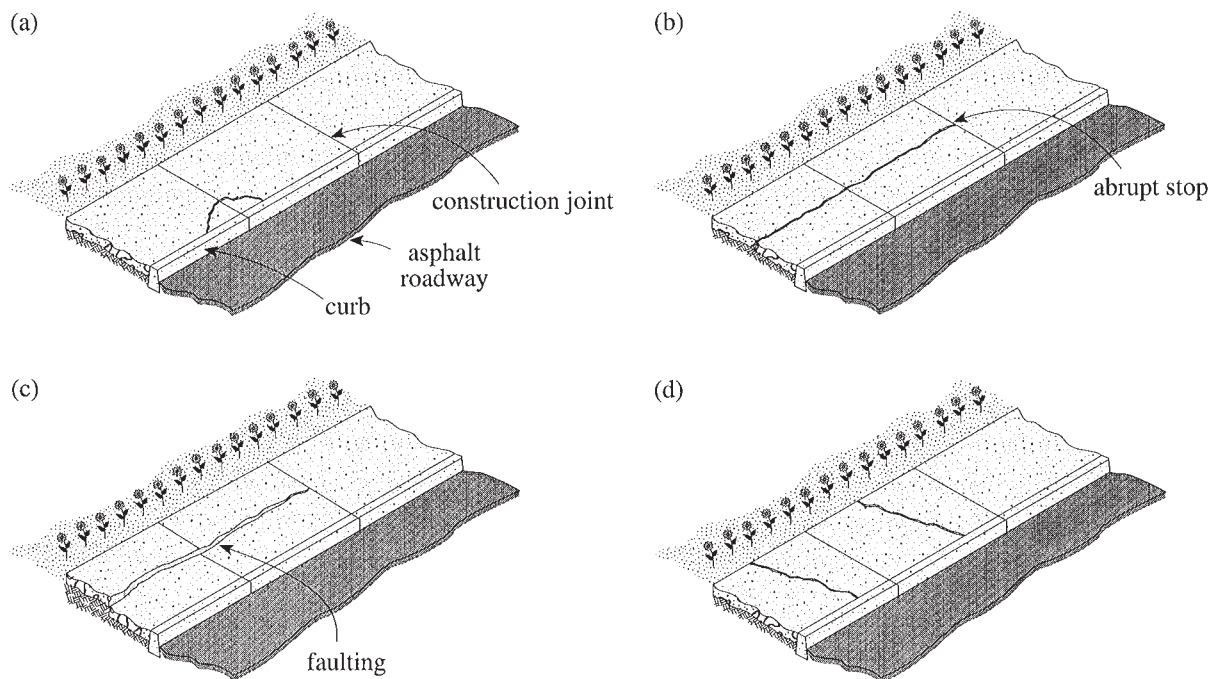
Fig. 1. Typical characteristics of sidewalk cracking patterns: (*a*) "D" corner cracks; (*b*) longitudinal cracks, no faulting; (*c*) longitudinal cracks, with faulting; (*d*) transverse cracks.

widespread type of sidewalk cracking is longitudinal (Figs. 1*b* and 1*c*). These cracks usually occur at the centre of the sidewalk and can extend through several expansion joints before stopping abruptly. If the cracks do not open up, and if no faulting develops along these longitudinal cracks, they do not pose a safety hazard. However, this type of crack normally opens up with time and it is not uncommon to observe faulting of the order of 10 mm. Transverse sidewalk cracking (Fig. 1*d*) is also generally considered to be a major concern especially if a significant tripping edge (faulting) develops.

Concrete sidewalks are essentially slabs-on-grade except that their typical widths are less than 1.2 m. The behaviour of sidewalks is dependent on the ongoing interaction with the soil on which it rests. Thus, the behaviour of sidewalks is best understood by studying the corresponding soil–structure interaction rather than the individual aspects of the structural concrete slab or the soil characteristics.

In collaboration with the cities of Winnipeg, Saskatoon, Regina, Edmonton, Calgary, and Camrose, the National Research Council Canada initiated a project to identify the underlying mechanisms that lead to longitudinal cracks in sidewalks. The different tasks within this project were (*i*) to conduct a field engineering survey of selected areas of these five major cities to document the extent and type of damage, (*ii*) to observe and analyse the behaviour of sidewalks by taking regular surface elevation measurements, (*iii*) to conduct small-scale laboratory tests to replicate field observations, and (*iv*) to develop analytical and numerical methods that model the behaviour of sidewalks. This paper addresses only the first two tasks and subsequent papers will address the other tasks.

Sidewalk construction practice

A concrete sidewalk slab is typically 150 mm thick and a

common configuration of sidewalk construction is that the distance between construction joints is marginally larger than the width of the sidewalk. Before the mid seventies, sidewalks were essentially constructed of two components, a flat concrete slab of uniform thickness adjacent to an L-shaped beam that forms the curb and gutter. These two components are usually separated by a full-depth joint. More recently, the preferred design is without any full-depth joints between the flat slab, curb, and gutter. The monolithic sidewalk construction method improves productivity and vehicle accessibility from the roadway to garages. Continual visual monitoring of both types of sidewalks over the past few years indicates that both types of sidewalks are equally prone to cracking and that no benefit in terms of performance seems to be gained from either method of construction.

Typically, concrete sidewalks in new housing developments are built using an extrusion technique where the operations of grading of the native soil and its compaction to 95–98% Proctor density and casting of concrete are done in one single pass. Since the late 1980s, replacement sidewalks have been hand poured and a 150 mm granular subgrade has been provided directly beneath the sidewalks. Some municipalities provide transverse or longitudinal steel reinforcement but the practice is not always uniform nor consistent.

Field observations of sidewalk damage

It is generally perceived that concrete sidewalks in most major urban areas in the Canadian prairie provinces have extensive damage. Yet, there is little documented evidence of the extent of damage or the predominant failure modes. It is well known that the surficial soils within many parts of this region are silty fine-grained soils which are particularly prone to frost heave.

Fig. 2. Failure modes of concrete sidewalks: (*a*) rigid body movements; (*b*) Shattering.
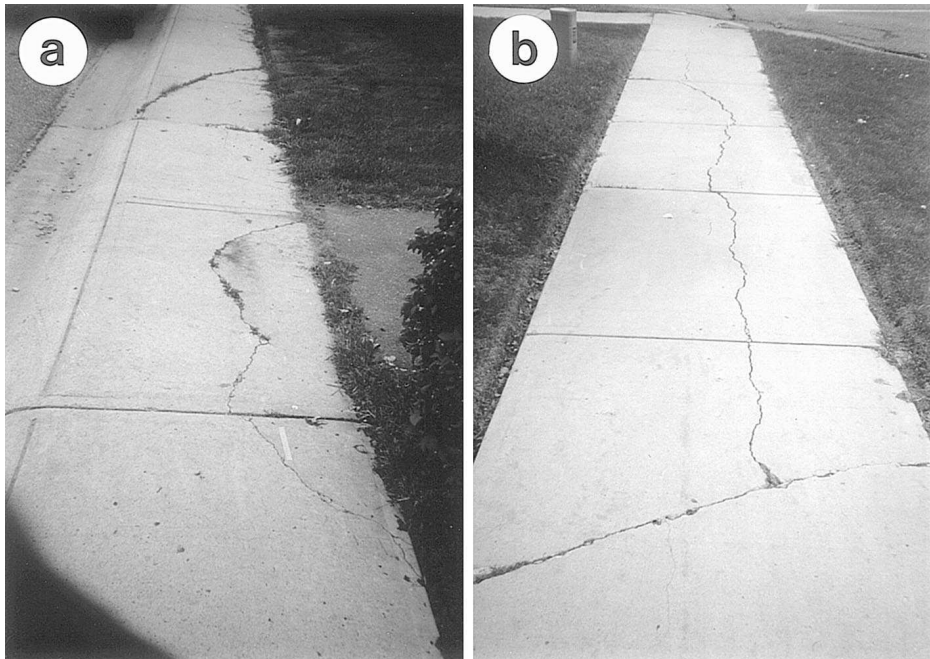

Table 1. Extent and type of sidewalk damage in five cities.

City	Mixed mode* (%)	Longitudinal (%)	Transverse (%)	No damage (%)
Winnipeg	23	26	20	30
Saskatoon	42	4	49	5
Regina	16	35	13	37
Edmonton	60	10	18	10
Calgary	45	7	35	13

*Both longitudinal and transverse cracks with longitudinal cracks forming the majority.

Though both the prairie provinces and parts of eastern Canada are overlain by frost susceptible soils, severe damage to concrete sidewalks manifests itself mainly in the prairie provinces.

The weather profiles of the prairie provinces and areas around the St. Lawrence River Valley are different. For example, the mean freezing indices for Edmonton and Ottawa are 1800°C·d and 1000°C·d, respectively. In general, the climate in the Great Lakes and coastal areas of Ontario and Quebec is humid, and water tables are relatively shallow. Thus soil moisture contents are high. By contrast, the prairie provinces are semi-arid to arid areas and acute soil-moisture deficiencies can develop during extreme dry climatic periods.

During the summers of 1993 and 1994, selected areas of each major city were visually surveyed to record the extent and type of damage to concrete sidewalks. Surficial soil samples were taken at locations whether there was sidewalk damage or not. Soil samples taken at depths of 200–300 mm below the ground surface were primarily used to identify the soil and to determine soil index properties. The observed sidewalk damage (Table 1) was mapped together with the geographical distribution of index properties of surficial soils (Table 2) within each neighbourhood. The data collected from each city (Rajani 1995) are too numerous to present in this paper, therefore specific illustrative examples are provided which are valid for all five major cities.

Damage to sidewalks can occur in one of several ways, i.e., rigid body movements (Fig. 2*a*), joint buckling/blow-up, popouts, shattering (Fig. 2*b*), corner break, scaling, crazing (Fig. 3*a*), shrinkage cracks, longitudinal cracks (Fig. 3*b*), faulting, "D" cracks (Fig. 4*a*), transverse cracks (Fig. 4*b*), joint spalling, or corner spalling. Popouts, shattering, corner breaks, scaling, crazing, shrinkage cracking, joint spalling, and corner spalling occur in concrete sidewalks independent of the geographic location and even outside the regions under study here. These damages result from the use of either poor or inappropriate types of concrete. Transverse or "D" cracking takes place primarily due to uneven or non-uniform subgrade compaction rather than an overall lack of compaction effort. It is also worth noting that "D" cracking also occurs in regions where soils are not particularly frost susceptible. This type of damage in concrete sidewalks is also observed in many urban areas in Canada. However, extensive longitudinal cracks in sidewalks occurs primarily in the prairie provinces. These longitudinal cracks are usually within the middle third of the sidewalk.

Typically, the longitudinal and transverse cracks had average widths of 5–10 mm and fault heights of 3–8 mm. Nonetheless, sidewalk cracks as wide as 30 mm and fault heights as high as 32 mm were observed. While there is the perception that longitudinal sidewalk cracks occur alone, the surveys showed that usually these longitudinal cracks are present together with transverse cracks.

All five cities have surficial clays with low, medium, and high plasticity (Fig. 5). In particular, Regina appears to have predominantly high compressible inorganic silts. The range of plasticity index (11–36%) and clay activity indicates that the potential for volume change as a result of swelling is low to medium. The activity (ratio of plasticity index and percent by

Table 2. Index properties for surficial soil in five cities.

City	Liquid limit (%)		Plastic limit (%)		Plasticity index (%)	
Winnipeg	60	(35.8 to 76.6)	30.8	(16.6 to 47)	29.2	(19.2 to 29.6)
Saskatoon	35.2	(25 to 46)	24.1	(21 to 28)	11.1	(4.0 to 18.0)
Regina	62.4	(34.4 to 77.1)	26.6	(15.5 to 34.1)	35.8	(18.9 to 43.0)
Edmonton	51.6	(33.3 to 66.3)	24.5	(18.1 to 32.5)	27.1	(15.2 to 33.8)
Calgary	34.6	(23.5 to 51)	21.9	(15 to 35)	12.7	(8.5 to 16.0)

Note: The numbers in parentheses indicate the range of values.

Fig. 3. Failure modes of concrete sidewalks: (*a*) crazing; (*b*) longitudinal crack along a curved sidewalk.
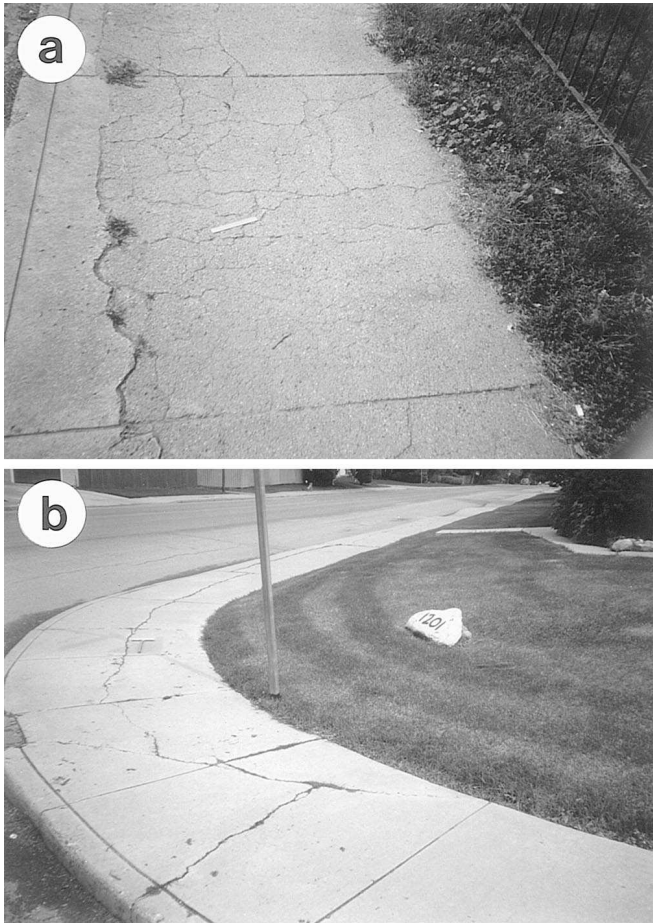
weight finer than 2 μm) of the clays as defined by Skempton (1953) varies on the average between 0.50 and 1.20, which categorizes these clays as inactive to normal. An attempt to correlate either the liquid limit or the plastic limit with the type of sidewalk damage in each city did not confirm any particular trends. However, a qualitative assessment indicates that the extent of longitudinal sidewalk damage increases (Fig. 6) with increasing plasticity index of the foundation soils. Soils with a high plasticity index imply that large changes in moisture are required for soils to go from a liquid state to a semisolid state. Pronounced changes in soil moisture are quite plausible, since extreme weather conditions are regularly encountered in the prairie provinces.

Concrete material properties

The observation that most observed concrete sidewalk cracks initially appear as hairline cracks is confirmation that forces acting on the concrete slab are large enough to induce stresses that exceed the tensile strength of concrete. It is well known that the tensile strength of concrete is limited and is usually in the range of 5–10% of the compressive strength. The tensile strength of concrete is usually reflected through the minimum strain at which a crack develops in concrete. Estimates for tensile failure strain of concrete are quoted (Shaker and Kennedy 1991) in the range of 200–300 με.

The flexural strain at which cracks initiate in concrete depends on the type and quality of concrete, sample size, and on environmental conditions. The maximum flexural strain (elastic part) can be estimated from the modulus of rupture and the elastic modulus. The modulus of rupture can be estimated from (Freedman 1985)

$$[1] \quad f_r = 0.75\sqrt{f_c'}$$

where f_r is the modulus of rupture in MPa and f_c' is the compressive strength of concrete in MPa. If the compressive strength of concrete is about 35 MPa, then the estimated modulus of rupture from the above equation is 4.4 MPa. The elastic modulus for concrete is about 20 GPa. As a result, the elastic flexural strain for concrete to crack under bending is about 220 με. However, the tensile stress at which concrete cracks for a particular concrete depends on a number of parameters (Collins and Mitchell 1991), such as the size of concrete member, strain gradients, and the restraint offered by the underlying soil at the bottom of the concrete slab. Consequently, the total failure strain for concrete sidewalks in bending is assumed to be between two and three times the elastic strain, i.e., a range of 440–660 με.

Failure modes of concrete sidewalks

The three deformation modes that can lead to sidewalk failure are summarized in Fig. 7. These failure modes are (*i*) sagging mode where either the centre of the sidewalk has a larger thaw settlement than at the edges, or instances where clays swell significantly at the edges; (*ii*) hogging mode where frost heave or upwards vertical movement due to swelling clays is greater at the centre than at the edges; and (*iii*) tensile-shrinkage failure mode where shrinkage of underlying soils induces tension in the concrete sidewalk. The tensile-shrinkage failure mode will be discussed in another paper. One way to confirm the deformation modes of sagging or hogging is to take vertical surface elevation measurements in the transverse direction at regular time intervals. As discussed above, the vertical surface

Fig. 4. Failure modes of concrete sidewalks: (*a*) "D" cracks; (*b*) transverse cracks.
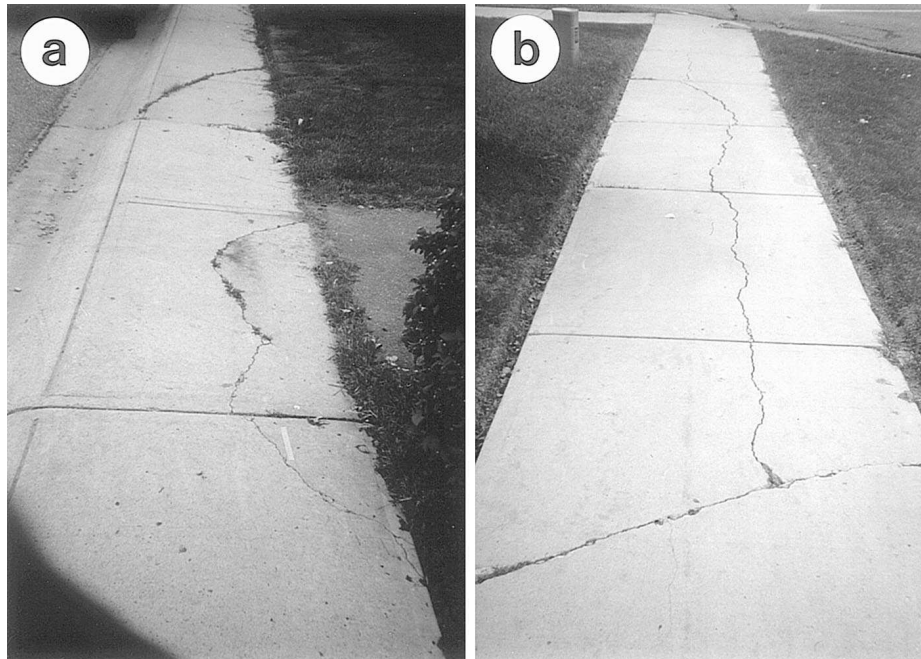
Fig. 5. Plasticity chart for surficial soils in prairie cities.
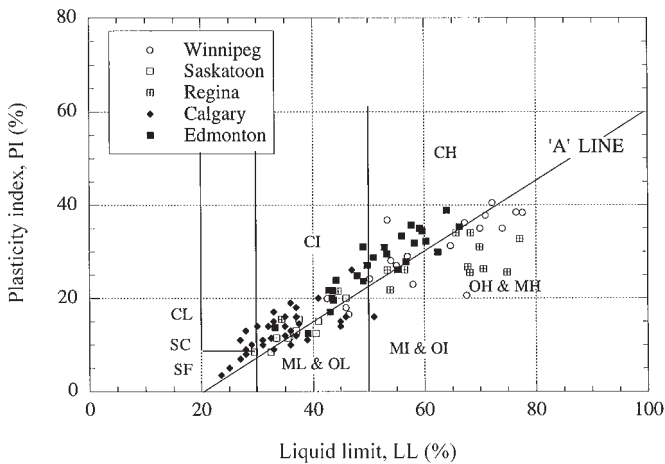
Fig. 6. Variation of extent of observed longitudinal sidewalk damage with plasticity index of surficial soils.
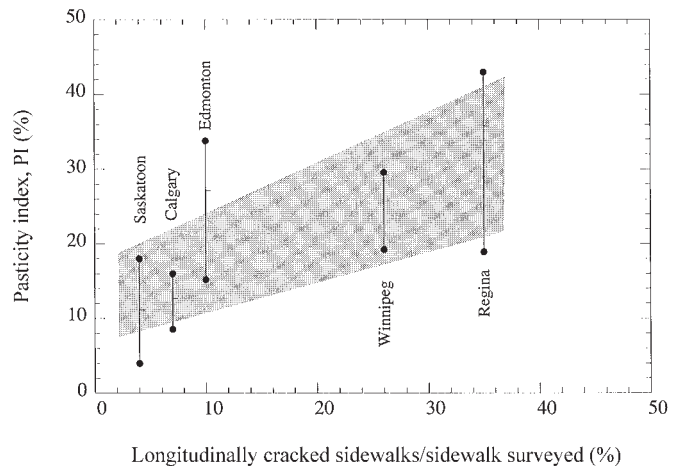

elevation measurements can also be used to obtain indirect estimates of flexural (bending) strains, which are useful to assess the impact of seasonal changes on the performance of concrete sidewalks.

In August 1993, all the five municipalities surveyed and the City of Camrose were invited to commence monitoring vertical movements of concrete sidewalks by measuring surface elevations at monthly intervals across transverse sections of sidewalks, at two locations in each city. These measurements are typically taken at approximately 150 mm intervals with a minimum of six points across the width of the sidewalk. The measurements are taken once every month. The points where these elevations are obtained are permanently marked, say, with paint or steel studs to ensure that successive elevations are always measured at the same points. It is estimated that the surface elevation measurements can have an error of ±1 mm.

The implications of these measurement errors will be discussed later. Edmonton, Calgary, and Camrose took these vertical surface elevations measurements over a period of 2 years. These data are analyzed and discussed in this paper.

The data on vertical surface elevations need to be processed appropriately to gain an insight on the modes of deformation and to obtain indirect estimates of flexural (bending) strains. As a first step, the non-damaging modes of movement, i.e., rigid body movements, need to be extracted from the total measured movements (Fig. 8). The next step is to identify numerical methods that are best suited to obtain good estimates of flexural strain from differential vertical movements. A number of methods can be used to analyze the vertical movement of sidewalks, but not all methods are well suited to analyze concrete sidewalks because of their sensitivity to

Fig. 7. Possible failure modes of sidewalks: (*a*) tensile-shrinkage; (*b*) sagging; (*c*) hogging.

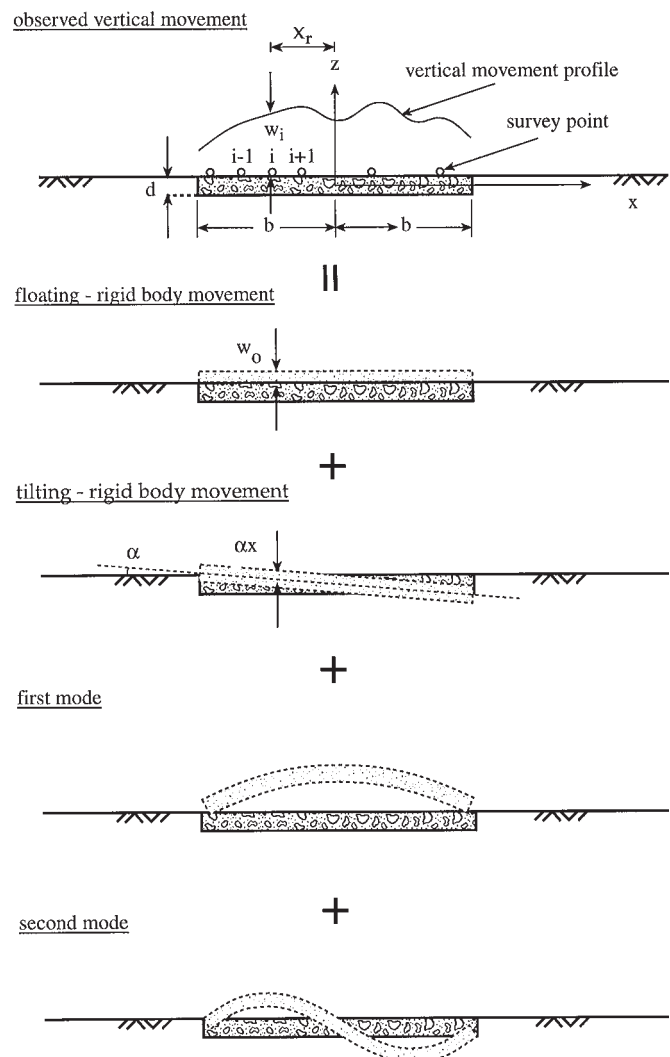**Fig. 8.** Components of vertical movement for a typical sidewalk slab.
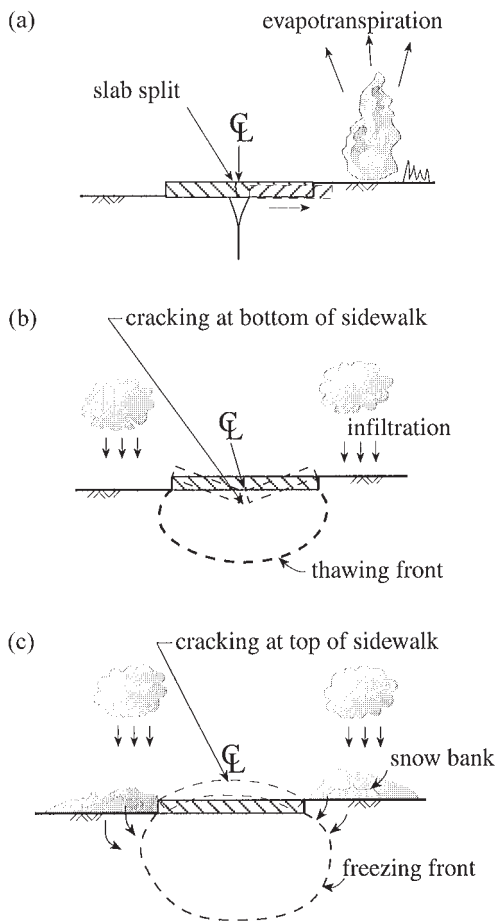
measurement errors. Four possible methods, finite difference, cubic spline, polynomial fit, and Fourier series methods, were examined to obtain estimates of flexural strains (Zhan and Rajani 1995). The Fourier series method with one sine term was identified as the most suitable method. Histories of rigid body movements and flexural strains are determined to assess their association with changes in climatic conditions.

Analysis of sidewalk vertical movements

The data on vertical movement of sidewalks at Edmonton, Calgary, and Camrose were analysed using the Fourier series smoothing procedure. The rigid body movements (uniform vertical movement and tilt) and flexural strains were determined at the end of each 30-day period.

Figure 9 includes the comparisons of uniform vertical movements with time at all the sidewalk locations, two from each city, as well as the variation of frost depth with time at Twin Brooks in Edmonton (one of two locations in Edmonton). Except for the sidewalk at Parkview II in Camrose, all sidewalks showed similar time dependence of the uniform vertical movements. The upward sidewalk movement began in November, 1993, reached a maximum vertical movement value around March, 1994, and then lowered down to a more or less zero vertical movement position in May, 1994. This time period also corresponds to cold seasonal temperatures in

Canada. Both uniform vertical movement (Fig. 9) and frost depth (Fig. 10) had similar time dependence at the two sidewalks in Edmonton (Zhan and Rajani 1995) where the frost penetration and soil moisture changes were monitored. Consequently, it can be inferred that the uniform vertical movement is primarily caused by the frost penetration beneath the sidewalk. The insignificant variation of the uniform vertical movement during the warm season is an indication that the uniform vertical movement is not very sensitive to the moisture change. The maximum uniform vertical movement for the sidewalk at McLeod was about three times of that for the sidewalk at Twin Brooks, although the soil under the sidewalk at McLeod, Edmonton, had an average moisture content of about 13% less than the soil under the sidewalk at Twin Brooks, Edmonton (Fig. 10). This observation provides evidence that a large variation in soil moisture in the same geographical location is to be expected.

The variations of the tilt angle (rotation) with time for sidewalks are shown in Fig. 11. The sidewalk usually started to tilt in November and peaked in March, which is similar to the

Fig. 9. Comparison of uniform vertical movements with time.
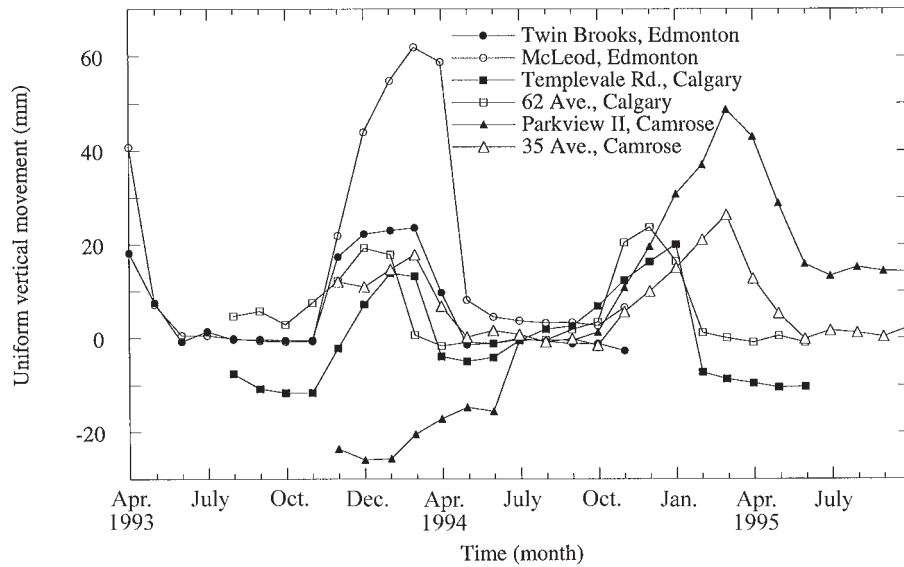
Fig. 10. Variation of frost depth and soil moisture with time.
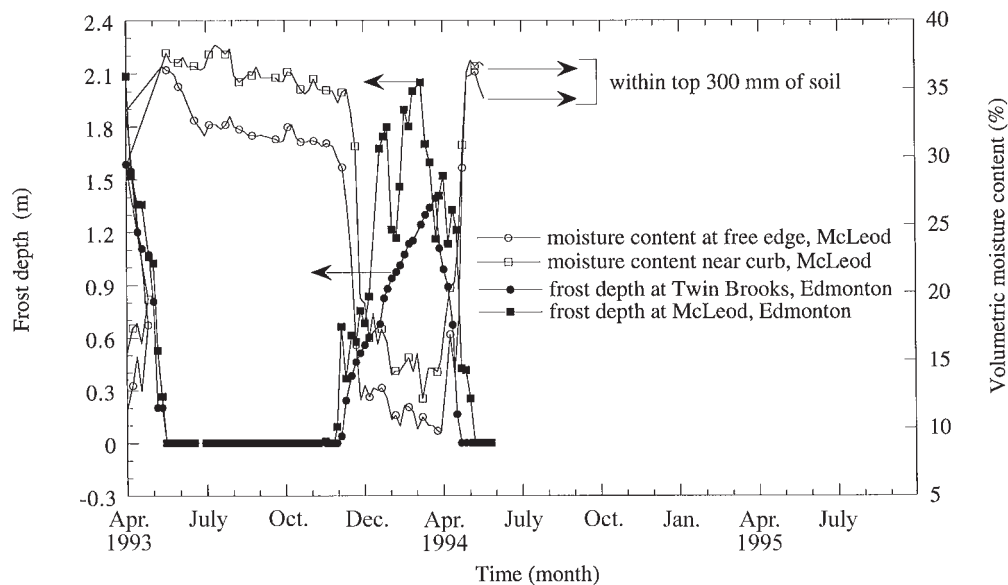
observed seasonal uniform vertical movements. All the sidewalks except the one at Twin Brooks, Edmonton, tended to tilt away from the road. Moisture content measured at McLeod, Edmonton, indicated higher moisture content for the soil close to the road, which may explain greater frost heave at this side (Fig. 10). The tilt angles were always less than 1°. As explained earlier, the uniform vertical movement and tilt components of rigid body modes of movement are non-damaging and any specific values have no particular significance.

The variation of flexural strains at the centre of the sidewalk with time is shown in Fig. 12 for all the sidewalks. Some time between January and February 1993, a longitudinal crack developed a few metres away from the line of measurements at the Twin Brooks sidewalk location in Edmonton. On the other hand, the sidewalks at Templevale in Calgary and 35 Avenue

in Camrose had already cracked before the field measurements started. In vertical movement analysis it was assumed that the sidewalks were uncracked and, as a result, the estimated strains reflect the presence of cracks in the sidewalks. The strains in these two sidewalks are large in comparison with estimates of strain from other locations and far exceed the admissible tensile strain for concrete. The next sidewalk likely to crack is the one at 62 Avenue in Calgary. However, the flexural strains for all other sidewalks are appreciably less than the 440–660 $\mu\varepsilon$ range. It is difficult to discern how the flexural strains vary with seasonal conditions. An alternative form of representing the flexural strain is to normalize the calculated strain with the maximum absolute value. These variations of the normalized flexural strains with time at the centre of the sidewalk are shown in Fig. 13. In general, the normalized flexural strains

Table 3. Maximum and minimum values for uniform vertical movement, tilt, and flexural strains.

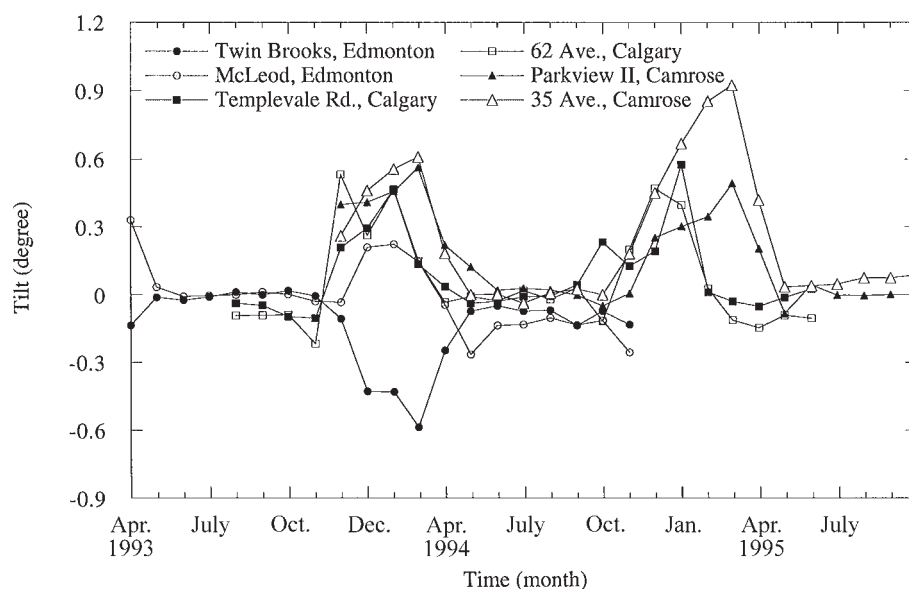| Location	Uniform vertical movement (mm)		Tilt (deg)		Flexural strain (με)	
	Min.	Max.	Min.	Max.	Min.	Max.
Edmonton						
Twin Brooks	−3.0	24.0	−0.6	0.0	−456	213
McLeod	−0.7	62.0	−0.3	0.3	−117	625
Calgary						
135 Templevale	−11.6	20.0	−0.1	0.6	−3300	225
8412–62 Ave.	−1.7	24.0	−0.2	0.5	−599	105
Camrose						
Parkview II	−26.0	43.0	−0.1	0.6	−390	371
6315–35 Avenue	−1.5	26.0	−0.0	0.9	−3375	424

Fig. 11. Comparison of tilt angle (rotation) with time.

Fig. 12. Comparison of flexural strains with time.
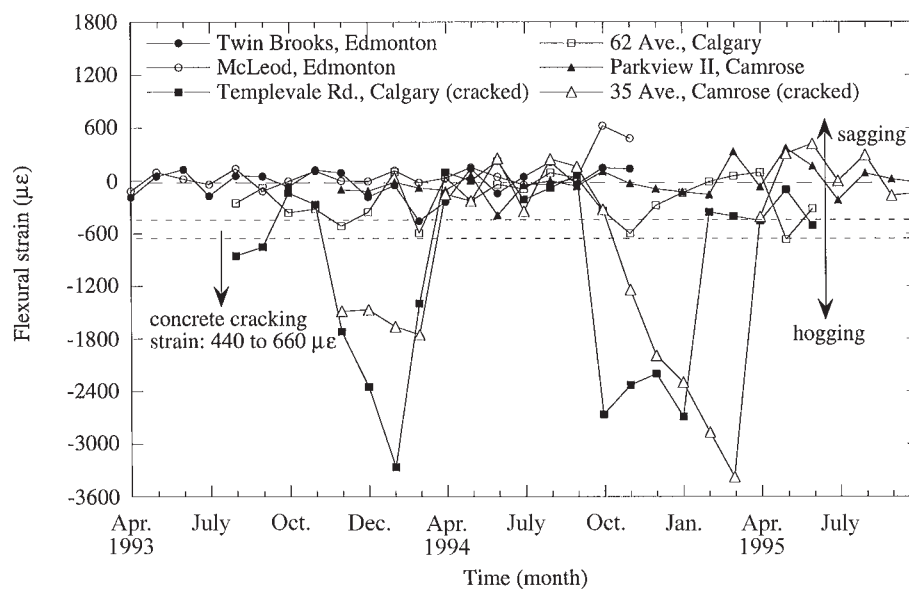
Fig. 13. Comparison of normalized flexural strains with time.
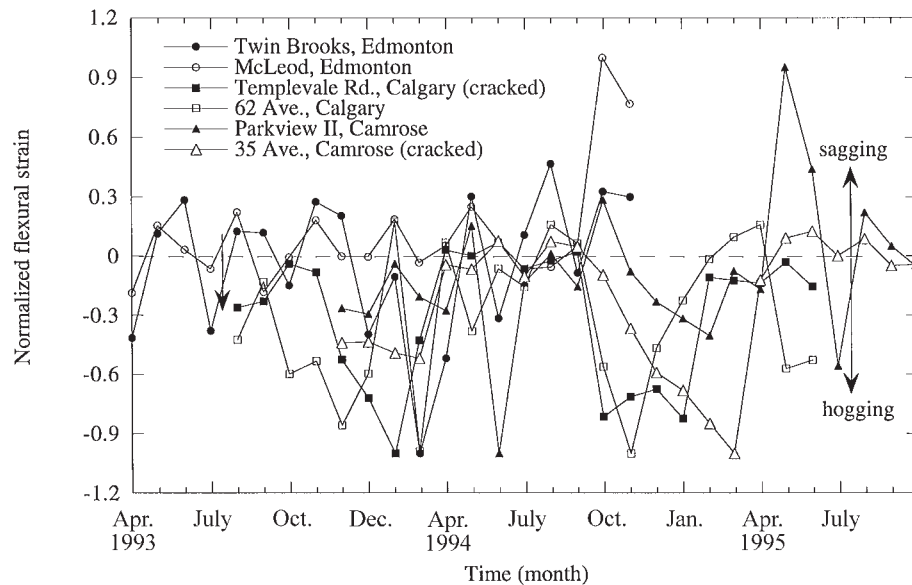
Fig. 14. Comparison of differential movement ratios with time.
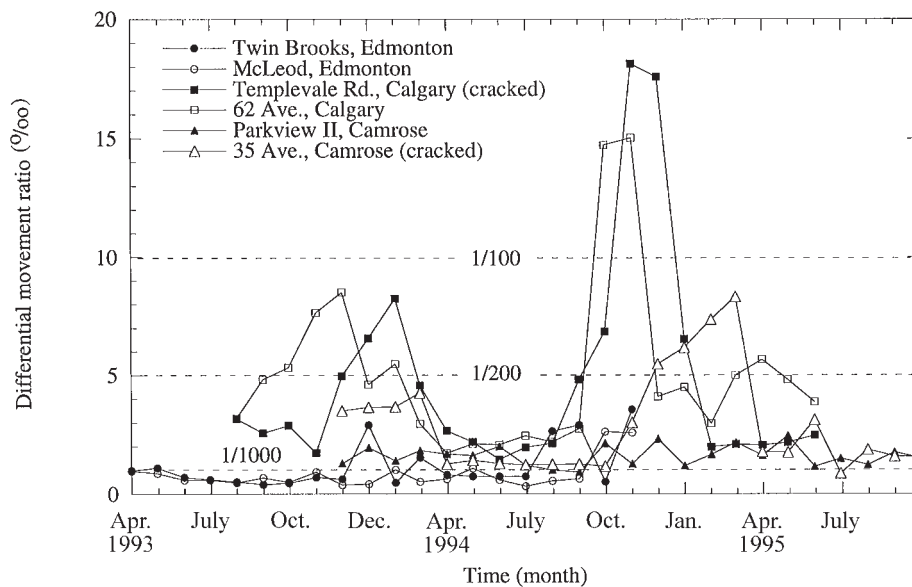
are higher during the cold season (November to April) than during the warm season. Furthermore, normalized flexural strains for the majority of the sidewalks are negative, which corresponds to the hogging mode of deformation.

The maximum and minimum values for uniform vertical movement, tilt, and flexural strain are summarized in Table 3 for all sidewalk locations. Although the uniform vertical movement for the sidewalk at McLeod, Edmonton, is much larger than at other sidewalks, the resulting flexural strain is much smaller. This is a clear indication that only the differential vertical movement is the damaging component of vertical movements.

In lieu of admissible flexural strains, admissible total settlement, tilting, and differential movement have been established for a wide variety of structures (Lambe and Whitman 1969; Nelson and Miller 1992). The most common method of

expressing admissible movement is that the ratio of differential movement (angular distortion) to span of the structure should not exceed a certain value established on the basis of engineering judgment and experience with the behaviour of structures. The established allowable ratios of differential movement to span for the slab-on-grade have been derived largely from the practice of limiting damage to superstructures. The range of the ratios is

$$[2] \qquad \frac{1}{1000} \le \frac{W_d(\text{max})}{2b} \le \frac{1}{200}$$

The range of allowable differential movement ratios is shown in Fig. 14 together with the measured ratios obtained at all sidewalk locations. While allowable differential movement ratios always exceed 1/1000, notable peaks occur during the cold season. The maximum estimated flexural strains for sidewalks

at Templevale Road in Calgary and 35 Avenue in Camrose coincide with the peak values of allowable differential movement ratios. Thus, an alternative criterion to limit damage to slab-on-grade is to ensure that the allowable differential movement ratio is less than 1/200.

Summary and conclusions

Different failure modes of concrete sidewalks have been identified and validated through an extensive field survey of sidewalk inventories in five major cities in the Canadian Prairies. The major form of observed sidewalk damage is longitudinal cracks. All five cities have surficial clays with low, medium, and high plasticity. The range of plasticity index (11–36%) and clay activity indicate that the potential for volume change as a result of swelling is low to medium. An attempt to correlate either the liquid limit or the plastic limit of the soil with the type of sidewalk damage in each city did not confirm any particular trends. However, a qualitative assessment indicates that the extent of longitudinal sidewalk damage increases when the sidewalks are founded on soils with a high plasticity index.

Two of the three failure modes (hogging and sagging) are closely examined through the analysis of observed vertical surface movements of sidewalks at several locations in different cities. The analyses of vertical movements of sidewalks showed that the rigid body movements (both uniform vertical movement and tilt) are mainly a consequence of frost penetration beneath the sidewalk. Uniform vertical movement is not very sensitive to moisture changes in the soil underneath the sidewalk. The dominant mode of deformation for all the sidewalks is hogging, which is accentuated in the cold season. Normalized flexural strains and ratios of differential movement were found to be good indicators for limiting damage to slabs-on-grade.

The predominant hogging deformation mode during the cold season can be mitigated by the use of rigid insulation boards beneath the sidewalk. The intent of the insulation board is to promote uniform vertical movement and minimize differential movement. However, the insulation boards should extend well beyond the edges of the sidewalk to promote equal frost penetration across the width of the sidewalk. Another possible mitigative measure is to place a coarse granular 150 mm thick subgrade below the sidewalk. This coarse granular subgrade ensures good drainage and hence minimizes the unequal accumulation of soil moisture, which has direct bearing on the frost susceptibility of the underlying soils. The coarse granular subgrade also ensures that high suction pressures do not develop beneath the sidewalk and thus avoid tensile-shrinkage failure of the sidewalk. A granular material properties such as light weight aggregate manufactured from expanded shale in a rotary kiln has good thermal insulation and drainage properties which makes it ideal for use as a subgrade for sidewalks.

Acknowledgments

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List of symbols

b half-width of the concrete sidewalk slab

d thickness of the concrete sidewalk slab

$f_{r,}$ modulus of rupture in MPa

f_c' compressive strength of concrete in MPa

W_d differential vertical movement

W_i vertical movements at point i

W_o first rigid body mode of movement or uniform vertical movement

X_r coordinate of the survey point

α second rigid body movement, i.e., tilt