NRC Publications Archive Archives des publications du CNRC

Quantitative analysis of ice sheet failure against an inclined plane Frederking, R. M. W.; Timco, G. W.

This publication could be one of several versions: author's original, accepted manuscript or the publisher's version. /
La version de cette publication peut être l'une des suivantes : la version prépublication de l'auteur, la version acceptée du manuscrit ou la version de l'éditeur.

Publisher's version / Version de l'éditeur:

Proceedings of the Fourth International Offshore Mechanics and Arctic Engineering Symposium, 2, pp. 160-169, 1985-02-10

NRC Publications Archive Record / Notice des Archives des publications du CNRC :

<https://nrc-publications.canada.ca/eng/view/object/?id=2e24db67-1013-47bb-ac0d-bd5ad4a57d95>
<https://publications-cnrc.canada.ca/fra/voir/objet/?id=2e24db67-1013-47bb-ac0d-bd5ad4a57d95>

Access and use of this website and the material on it are subject to the Terms and Conditions set forth at
<https://nrc-publications.canada.ca/eng/copyright>

READ THESE TERMS AND CONDITIONS CAREFULLY BEFORE USING THIS WEBSITE.

L'accès à ce site Web et l'utilisation de son contenu sont assujettis aux conditions présentées dans le site
<https://publications-cnrc.canada.ca/fra/droits>

LISEZ CES CONDITIONS ATTENTIVEMENT AVANT D'UTILISER CE SITE WEB.

Questions? Contact the NRC Publications Archive team at
PublicationsArchive-ArchivesPublications@nrc-cnrc.gc.ca. If you wish to email the authors directly, please see the first page of the publication for their contact information.

Vous avez des questions? Nous pouvons vous aider. Pour communiquer directement avec un auteur, consultez la première page de la revue dans laquelle son article a été publié afin de trouver ses coordonnées. Si vous n'arrivez pas à les repérer, communiquez avec nous à PublicationsArchive-ArchivesPublications@nrc-cnrc.gc.ca.

Ser
TH1
N21d
no. 1285
c. 2
BLDG

13 556

National Research
Council Canada

Conseil national
de recherches Canada

QUANTITATIVE ANALYSIS OF ICE SHEET FAILURE AGAINST AN INCLINED PLANE

by R.M.W. Frederking and G.W. Timco

ANALYZED

Reprinted from
Proceedings of the Fourth International Offshore
Mechanics and Arctic Engineering Symposium, ASME
Dallas, Texas, February 10 - 15, 1985
Volume II, p. 160 - 169

DBR Paper No. 1285
Division of Building Research

Price \$1.00

OTTAWA

NRCC 24496

Canada

5498345

RÉSUMÉ

Les principales composantes de la force s'exerçant sur les structures inclinées construites au large des côtes dans l'océan Arctique comprennent le comportement de flexure et de broyage de la couverture de glace, la rotation et la translation de la glace fractionnée, la gravité et la flottabilité des glaces en remontée et en déplacement, ainsi que la friction entre la glace et la structure. Des essais ont été effectués afin d'isoler ces composantes, puis des expressions analytiques, en ont été déduites. Les éléments mesurés et prévisionnels se comparent avantageusement, le modèle analytique donnant un excès prévisionnel de l'ordre de 20%.

CISTI / ICIST

3 1809 00210 4849

QUANTITATIVE ANALYSIS OF ICE SHEET FAILURE AGAINST AN INCLINED PLANE

R. M. W. Frederking
Division of Building Research

G. W. Timco
Division of Mechanical Engineering
National Research Council of Canada

Ottawa, Ontario, Canada

ABSTRACT

The main force components on inclined offshore structures in the Arctic include the flexural and crushing behaviour of the ice sheet, rotation and translation of broken ice, gravity and buoyancy in lifting and moving ice, and friction between ice and structure. Tests were carried out to isolate these components and analytical expressions were derived for them. Measured and predicted components compare favourably, the analytical model over-predicting by about 20%.

INTRODUCTION

The interaction between a floating ice cover and an inclined structure is a complex process. A number of factors are at play, involving flexural and crushing failure of the ice sheet, rotation and translation of the broken ice pieces, gravity and buoyancy in lifting and moving the ice around the structure, and friction of the ice as it slides over the structure. These forces act intermittently for varying lengths of time. Their temporal and spatial sum represents the total ice force experienced by a structure.

Several investigators have analysed the problem of defining ice forces on structures with inclined faces (1-5). With the exception of Ralston's plastic limit analysis (5), all these approaches treat the ice as an elastic/brittle material. The overall interaction process between the ice cover and the structure is treated as either a two-dimensional or a three-dimensional problem.

Model investigations of ice forces on an inclined plane structure have also been carried out (6,7). Timco made an extensive comparison of model test results with Frederking's theory (3), showing that the theory consistently underpredicts by 0 to 40%, prompting a re-examination of the theory. An investigation of the load components was therefore carried out on isolated inclined structures. It involved experimental work to determine failure

sequence and processes and to define the force component in breaking and clearing of the ice sheet. The tests were performed as small-scale experiments rather than model tests, following certain similitude laws and scale factors. Analytical expressions were developed for the force components acting during failure of the ice and clearing of it over and around the structure.

THEORY

Development of the analytical model was based on observations of the failure process and measurement of force levels during tests with an inclined plane in an ice model basin. This was done by setting up an analogue chart recorder that could be observed simultaneously with the ice failure process. It was thus possible to relate the force levels with various parts of the failure sequence. Ice behaviour on an inclined plane is shown in Fig. 1, in which the ice cover is depicted as moving horizontally with respect to a fixed structure, as in nature. This is the point of view of the development, although for the actual tests performed in the model basin the structure was moved with respect to a fixed ice sheet. Three elements were identified as part of the total load generated by an ice cover moving slowly against an inclined plane. The first is from the breaking of the ice cover, the second from rotation of the broken pieces, and the third from sliding of the broken pieces up the inclined plane. The spatial and temporal sum of the forces acting on the structure can be represented by a vertical force, V , and a horizontal force, H , resolved from the normal, N , and shear, S , forces (see Fig. 1). The relation between the two sets of components is

$$\begin{aligned} H &= N \sin \alpha + S \cos \alpha \\ V &= N \cos \alpha - S \sin \alpha \end{aligned} \quad (1)$$

where α is the angle of inclination from the horizontal. Assuming classical Coulomb friction,

FIG. 1 Schematic of ice behaviour on an inclined plane structure

$S = \mu N$, where μ is the coefficient of friction. The ratio of the horizontal to vertical force, ξ , is defined as

$$\xi = \frac{H}{V} = \frac{\sin \alpha + \mu \cos \alpha}{\cos \alpha - \mu \sin \alpha} \quad (2)$$

Each of the force components will be considered in sequence.

Breaking

A significant development during the model tests was the observation that maximum breaking load was associated with formation of radial cracks in the ice in front of the inclined plane rather than with formation of circumferential cracks. The actual process was as follows: (a) load increased rapidly as the ice cover moved against the inclined plane, (b) load peaked as radial cracks formed in ice at the centre and edges of the plane, (c) load dropped off to about half peak load and held steady for some time as the circumferential cracks formed. It had previously been assumed that radial cracks form at lower force levels and that the maximum failure load of an ice cover is associated with the formation of a circumferential crack. This, for example, was the approach used by Frederking (3). The following analysis of the failure component will be based on the load required to produce radial cracks, in accordance with the observed failure behaviour.

To verify the observed failure load behaviour two special tests were conducted. An upward-breaking inclined plane with a slope angle of 45 deg was moved horizontally at a constant velocity against a straight ice edge. In one case two radial cracks at the edge of the plane were simulated by pre-cut slots. This resulted in failure in the form of a circumferential crack in the ice. In

the other case a circumferential crack was simulated by sawing an arc in the ice, thereby causing failure by radial cracking only. The two cases, together with measured vertical and horizontal loads on the structure, are illustrated in Fig. 2. The vertical failure load is 55 N for the radial pre-cuts versus 105 N for the circumferential pre-cut.

To calculate the ice breaking load based on the observed failure pattern, the process was mathematically modelled as a vertical line load on the edge of a semi-infinite ice sheet that is hydrostatically supported. This loading situation (shown schematically in Fig. 3) was also considered by Nevel (8), who performed a numerical integration to determine bending moment, M_x , at $x = 0, y = 0$. This initiates a radial crack that propagates in the positive y direction along $x = 0$. The mathematical simulation is not identical to the actual situation, but it is a reasonable approximation. The relation between the moment per unit length, M_x , and the length of the line load, B (equivalent to width of structure), is approximated by

$$\frac{M_x}{P} = 0.34 - 0.58 \log \left(\frac{B}{l} \right) \quad (3)$$

This expression was determined by fitting this simple equation to Nevel's (8) results for moment as a function of load length. The equation is a good fit for $B/l < 1$ and can be extrapolated up to about $B/l = 3$. The length of the line load is non-dimensionalized by dividing by the characteristic length of the ice cover, l

FIG. 2 Model test results for a 45-deg upward-breaking inclined plane. $h = 38.5$ mm, velocity of structure = 3 mm/s

FIG. 3 Uniform line load, p , at edge of semi-infinite ice sheet

where $\ell = \left(\frac{Eh^3}{12k(1-\nu^2)} \right)^{1/4}$ (4)

E = elastic modulus of ice
 h = ice thickness
 k = foundation modulus
 ρg (ρ = density of water, g = gravitational acceleration)
 ν = Poisson's ratio = 1/3.

The upper limit on the vertical load, P , is reached when M_x equals the bending moment capacity of the ice sheet. Bending moment capacity is usually determined from small in situ cantilever beam tests in the ice cover and the results of these tests are given in terms of flexural strength. The relation between flexural strength, σ_f , and bending moment capacity, M_o/b , is

$$\sigma_f = \frac{6 M_o}{b h^2} \quad (5)$$

where b is the width of the beam. For a given loading length, B , the maximum vertical load, P , is determined from the value of M_x/P in Eq. (3) and the flexural strength of the ice cover introduced through Eq. (5), with $M_x = M_o$. The bending moment capacity of a plate and a beam are not necessarily identical. Experiments by Lavrov (9) have shown that the moment capacity of a plate could be up to 25% greater than that of a beam. The analysis here however will assume that they are identical. On the other hand, due account has been taken of loading direction for flexural strength determinations since the strength with the top of the ice cover in tension is up to twice that for the bottom in tension. Combining Eqs. (3) and (5), the vertical load is given by

$$P = V_b = \frac{\sigma_f h^2}{2-3.5 \log \left(\frac{B}{\ell} \right)} \quad (6)$$

The test conditions of Fig. 2, $\sigma_f = 45$ kPa, $h = 38.5$ mm, $\ell = 440$ mm, $B = 1.0$ m, substituted in Eq. (6) yield a maximum vertical load of 90 N. By contrast, assuming pre-existing radial cracks for the test conditions above, the calculated vertical load to form a circumferential crack is 35 N, using the analysis of Frederking (3) and wedge angle $B/2 = 30$ deg. These calculated failure loads follow a pattern similar to the measured values in the special tests.

A method of calculating the vertical load to initiate formation of a radial crack has been outlined. This vertical load generates a corresponding horizontal load on the structure that

may be determined using Eq. (2). The maximum horizontal load due to breaking of ice, H_b , is thus

$$H_b = \xi \frac{\sigma_f h^2}{2-3.5 \log \left(\frac{B}{\ell} \right)} \quad (7)$$

Once radial cracks form, subsequent behaviour of the ice cover is that of a truncated wedge that fails by the formation of a circumferential crack. The location of the circumferential crack, i.e. breaking distance, ℓ_b , based on the analysis of Nevel (10) for a truncated cone, has been shown by Frederking (3) to be approximated by

$$\ell_b = 0.75 \ell \quad (8)$$

provided $B > 0.25 \ell$. Observations in the model basin indicated a similar breaking distance. As well, the horizontal load associated with circumferential breaking, H_{bc} , is given by (3)

$$H_{bc} = \frac{\xi \sigma_f h^2}{3} \tan(\beta/2) \left[(1.16 + 0.0325 \xi) + (2.26 + 0.0825 \frac{B}{2 \ell \tan \beta/2}) \right] \quad (9)$$

This includes a reduction in the breaking load due to the compressive stress generated in the ice cover. The radial and circumferential failures do not occur at the same time, but the larger of the two (the radial) was used in calculating total ice load.

Rotation

The next force component is that associated with the rotation of the broken ice slab until it is parallel to and lying on the top surface of the inclined plane. Frictional, gravitational, and buoyant forces are involved in this movement. The actual process is shown schematically in Fig. 4. The rotation of the slab, ϕ , is given by

$$\phi = \alpha - \arcsin \left\{ \sin \alpha \left(1 - \frac{U t}{\ell_b} \right) \right\} \quad (10)$$

Equation (10) indicates that ϕ increases from 0 to α as non-dimensional time $U t / \ell_b$ increases from 0 to 1. U is horizontal velocity and t is time. The gravity force, F_g , which acts vertically downward through the centre of gravity of the slab, $\bar{x}$, is independent of rotation, ϕ . To simplify the solution, buoyancy effects will be ignored; this is a conservative assumption that results in slightly higher predicted ice forces. Two reaction forces, N_r and μN_r , normal and parallel to the incline, act on the tip of the slab to rotate it. Summing the moments about 0, the following expression is obtained

$$F_g \bar{x} \cos \phi - N_r \ell_b \cos(\alpha + \phi_c - \phi) + \mu N_r \ell_b \sin(\alpha + \phi_c - \phi) = 0 \quad (11)$$

where

$$\bar{x} = \frac{\ell_b}{6} \frac{\frac{3B}{\ell_b} + 2 \tan \frac{\beta}{2}}{\frac{B}{\ell_b} + \tan \frac{\beta}{2}} \quad (12)$$

$$F_g = \gamma_i h A = \gamma_i h \ell_b^2 \left(\frac{B}{\ell_b} + \tan \frac{\beta}{2} \right) \quad (13)$$

FIG. 4 Schematic of geometric relations and forces acting on rotating ice slab (3)

FIG. 5 Variation of static rotation force factor $1/f(\phi)$

$$\phi_c = \tan^{-1} \frac{h}{\ell_b} \quad (14)$$

Solving Eq. (11) for the normal reactive force, N_r , developed by the structure, and simplifying

$$N_r = \frac{\bar{x} F_g}{\ell_b f(\phi)} \quad (15)$$

The variation of N_r as the slab rotates from the horizontal to an inclined position parallel to the slope is a function of ϕ and hence of time. A curve of $1/f(\phi)$ versus Ut/ℓ_b is presented in Fig. 5 for $\alpha = \pi/4$ (3). Both maximum and average forces can be determined from this variation. The horizontal force developed on the structure by slab rotation may be obtained by substituting Eq. (15) in Eq. (1)

FIG. 6 Sliding forces of ice on inclined plane

$$H_r = N_r \sin \alpha + \mu N_r \cos \alpha \quad (16)$$

Note that the maximum rotational force occurs at the beginning of rotation when $t = 0$.

Sliding

Once the broken ice is parallel to the inclined surface it slides up the slope, generating a normal and corresponding frictional force on the structure (Fig. 6a). The normal force due to sliding, N_s , is approximated by

$$N_s = F_s \cos \alpha \quad (17)$$

F_s is the weight of ice on the inclined plane

$$F_s = \gamma_i h B \left(\frac{Z}{\sin \alpha} + \frac{\ell_b}{2} \right) \quad (18)$$

where $Z/\sin \alpha$ is the slope length of the inclined plane, Z is the free-board of the structure, and $\ell_b/2$ is the maximum overhang of the ice slab before it topples off the top edge of the inclined plane. Substituting Eq. (17) in Eq. (1), with $N_s = N$ and $\mu N_s = S$, the horizontal sliding force, H_s , is

$$H_s = N_s \sin \alpha + \mu N_s \cos \alpha \quad (19)$$

The force, P , required to slide the ice up the slope also generates a reaction on the ice edge in contact with the structure. Figure 6b is a simplified representation of the forces acting in this area. The normal force generated on the structure as a result of this effect is

$$N_p = \frac{P \sin \alpha}{\cos \alpha - \mu \sin \alpha} \quad (20)$$

where

$$P = (\mu \cos \alpha + \sin \alpha) F_s \quad (21)$$

Substituting Eq. (21) in Eq. (1), with $N_p = N$ and $\mu N_p = S$, the additional horizontal sliding force, H_p , is

$$H_p = N_p \sin \alpha + \mu N_p \cos \alpha \quad (22)$$

This additional sliding force was neglected by Frederking (3). To obtain the total sliding force H_s and H_p are summed.

Computation

Having established the force components and written equations for each term, it is a simple matter to calculate the load components from the known mechanical properties of the ice and the characteristics of the structure. Determination of the total load, however, is complicated by the fact that the maximum value of each load component is not necessarily realized at the same instant. It will be assumed that an extreme total load can be determined by adding the maximum value of each component. This

is a very conservative assumption. These analytical expressions have value in that a single parameter can be varied independently in order to determine its effect on total load. In physical modelling this is difficult if not impossible because flexural strength, elastic modulus, and ice thickness are all interrelated.

To illustrate the effect of structure characteristics (B , Z , μ) and ice properties (σ , E , h) on the horizontal loads, a parametric study was run on the preceding equations. The results are presented in Fig. 7 and show the general trends predicted by the theory. The horizontal load on the structure shows a general, linear increase with increasing structure width, coefficient of friction and freeboard, structure width having the strongest influence of the three factors. Wedge angles, $\beta/2$, over the range 0 to 60 deg had no effect on the results, and a value of 30 deg was used. Ice thickness, along with strength and elastic modulus constant, has a very strong influence on load; in the range of ice thickness 3 to 8 cm load is proportional to $H^{1.5}$. On the other hand, flexural strength has a relatively weak effect on load, as do elastic modulus and characteristic length. They show, however, that as characteristic length, l , or elastic modulus, E , decrease below a particular value that maximum horizontal load increases. This is not a real effect but reflects on instability in Eq. (7) when the denominator approaches zero as B/l approaches 3.7.

EXPERIMENT

The tests were performed in the ice tank (21 m long by 7 m wide by 1.2 m deep) of the Hydraulics Laboratory, National Research Council of Canada, which uses urea model ice, and the structure was a painted plywood plane 1 m wide and inclined at 45 deg. The measured coefficient of friction between the model ice and the structure was 0.02. Two load cells were used to measure the vertical and horizontal loads on the structure; continuous load records were obtained and digitized at a frequency of 10 Hz, the mean and standard deviation subsequently being calculated. For comparison with predicted results the 95% confidence limit value (mean plus two standard deviations) was determined and is the value shown in parenthesis in the tables of results. All the tests were carried out at carriage speeds less than 3 cm/s in order to eliminate any inertial loading effects.

The first ice sheet, #33, was used to investigate the effects of various types of pre-cutting of the ice cover on measured horizontal and vertical loads. The results are summarized in Table 1, which also gives particulars of the structure and the ice sheet. A general trend of lower loads for the pre-cut ice sheets is evident. Disregarding test 3, for which a large accumulation of ice built up on the inclined plane, the loads for tests 4 and 5 were 20% lower and for test 2, where there was no ice breaking at all

FIG. 7 Effect of structure characteristics and ice properties on a 45-deg inclined plane structure

TABLE 1. Measured horizontal and vertical loads on 45-deg upward-breaking inclined plane structure - ice sheet No. 33

Structure characteristics		Ice characteristics
B = 100 cm		$\sigma_f = 85 \text{ kPa}$ + (downwards)
Z = 9 cm		$\sigma_f = 40 \text{ kPa}$ + (upwards)
$\mu = 0.02$		E = 139 MPa
Carriage speed = 2.7 cm/s		h = 4.2 cm
		l = 55 cm

Test No.	Pre-cuts ¹	Experimental Results, N
1	none	H = 167 ± 58 (283) ² V = 153 ± 49 (251)
2	2 parallel cuts 1 m apart and 30-cm spaced perpendicular cuts	H = 94 ± 39 (172) V = 98 ± 34 (166)
3 ³	2 parallel cuts 1 m apart and 15-cm spaced perpendicular cuts	H = 118 ± 106 (330) V = 115 ± 98 (311)
4	15-cm spaced perpendicular cuts	H = 106 ± 56 (218) V = 101 ± 48 (197)
5	2 parallel cuts 1.2 m apart	H = 132 ± 48 (228) V = 136 ± 42 (220)

¹Direction of pre-cuts is related to direction of horizontal motion of ice cover

²Maximum force, 95% confidence limit value (mean plus two standard deviations)

³Ice pieces folded in accordian fashion in front of inclined plane

TABLE 2. Model test results with 45-deg upward-breaking inclined plane structure - ice sheet No. 34

Structure characteristics		Carriage speed = 2.7 cm/s
B = 100 cm		
Z = 9 cm		
$\mu = 0.02$		

Test No.	Pre-cuts	Experimental Results, N
1	none $\sigma_f = 106 \text{ kPa}$ h = 4.1 cm E = 245 MPa l = 63 cm	H = 141 ± 66 (273) V = 132 ± 56 (244)
2	15-cm spaced perpendicular cuts 1.2 m long	H = 118 ± 67 (252) V = 113 ± 60 (233)
3	none $\sigma_f = 79 \text{ kPa}$ h = 4.2 cm E = 160 MPa l = 57 cm	H = 178 ± 58 (294) V = 160 ± 48 (256)
4	15-cm spaced perpendicular cuts 1.2 m long	H = 106 ± 49 (204) V = 101 ± 45 (191)
5	none $\sigma_f = 50 \text{ kPa}$ h = 4.2 cm E = 115 MPa l = 53 cm	H = 178 ± 62 (302) V = 161 ± 53 (267)
6	15-cm spaced perpendicular cuts 1.2 m long	H = 103 ± 48 (199) V = 97 ± 39 (175)

(ice pieces completely cut out), 40% lower than for the continuous ice cover.

Ice sheet #34 (Table 2) was tested for the cases of continuous ice cover and 15-cm spaced perpendicular pre-cuts for three different levels of flexural strength and elastic modulus of the ice. For continuous ice cover, horizontal and vertical loads increased with decreasing flexural strength and elastic modulus, whereas for pre-cuts the loads decreased with decreasing flexural strength and modulus. The differences are quite small, less than 10%, and not considered significant given the natural scatter in such testing. Timco (7) found about a 10% increase in total horizontal load as flexural strength increased from 15 to 45 kPa.

For ice sheet #35 (Table 3) different patterns of pre-cutting were again tried, but in this case with a much larger freeboard (Z) on the structure than for the previous two ice sheets. The perpendicular pre-cuts, tests 2 and 3, resulted in slightly higher loads than those from the continuous ice cover, test 1. This was most probably due to the greater bulk of ice piling up on the face of the structure, where the ice had perpendicular pre-cuts. For parallel pre-cuts the load was reduced by about a third.

Ice sheets #36 and #37 (Tables 4 and 5) were used primarily to examine the ice failure process and breakings loads. This has already been discussed. The results of a number of slow-rate (0.3 cm/s) loading tests to examine breaking loads are summarized in Table 4. In the case of radial pre-cuts failure was by formation of a circumferential crack and in the case of a circumferential pre-cut failure was by radial cracks formed under flexural stress conditions. In no case was there a failure mode that could be described as shear. They show that for radial pre-cuts (tests 2 and 3) the breaking loads are significantly less than for the circumferential cuts (test 4), or none (test 5); interestingly, these are quite similar to each other. For ice sheet #37

(Table 5), which was thicker and stronger than sheet #36, the breaking load associated with radial pre-cuts was about a third of the breaking load of an unaltered or circumferential pre-cut ice cover.

Taking the test conditions of ice sheets #33 to #37 and the analytical model developed earlier in the paper, loads can be predicted. The results of a number of such predictions are present in Table 6. Note that rotation and sliding (33-2), and breaking and rotation, have been combined, since they could not be experimentally separated. These predicted values are plotted versus corresponding measured values in Figure 8. For almost all the cases plotted the predicted values are greater than the experimental loads.

DISCUSSION

The physical model tests provide insight into the failure behaviour of an ice cover acting on an inclined structure. Observations of failure processes show that the maximum load associated with breaking of an ice cover is related to the formation of radial cracks in the ice in front of the structure. This was confirmed by measurements of failure loads of ice covers pre-cut so as to induce failure by radial or circumferential crack formation. An existing analytical model (3) has been modified as a result of this failure behaviour. The model identifies rotation and sliding of the broken pieces as additional load components. While there is reasonable agreement between experiment and the analytical model in most cases investigated the fact that there are a few instances where agreement is not good suggests that there is still need for further work in refining the analytical model.

One of the first areas to be investigated is that of failure of the ice cover. The structure has been simulated as a line load; a closer representation of the action of the structure on the ice, however, is that of a line displacement. As this representation

TABLE 3. Model test results with 45-deg upward-breaking inclined plane structure - ice sheet No. 35

Structure characteristics		Ice characteristics
B = 100 cm		$\sigma_f = 90 \text{ kPa} \pm$
Z = 26 cm		$\sigma_f = 52 \text{ kPa} \pm$
$\mu = 0.02$		$E = 153 \text{ MPa}$
		$h = 4.3 \text{ cm}$
Carriage speed = 2.7 cm/s		$l = 57 \text{ cm}$
Test No.	Pre-cuts	Experimental Results, N
1 ¹	none	H = 233 $\pm$ 65 (363) V = 213 $\pm$ 56 (325)
2	20-cm spaced perpendicular cuts 1.2 m long	H = 233 $\pm$ 82 (397) V = 222 $\pm$ 72 (366)
3 ²	20-cm spaced perpendicular cuts 1.2 m long; 15-cm long parallel cuts at either end of perpendicular cuts	H = 225 $\pm$ 76 (377) V = 222 $\pm$ 71 (364)
4	2 parallel cuts 1.2 m apart	H = 161 $\pm$ 36 (233) V = 157 $\pm$ 38 (233)

¹Ice thickness for this test was 4.1 cm

²Ice piled up in front of structure, accumulating 2-3 layers on plane

TABLE 4. Model test results with 45-deg upward-breaking inclined plane structure - ice sheet No. 36

Structure characteristics		Ice characteristics	
B = 100 cm		$\sigma_f = 45 \text{ kPa} +$	
Z = 18 cm		$\sigma_f = 23 \text{ kPa} +$	
$\mu = 0.02$		E = 74 MPa	
		h = 3.85 cm	
		l = 44 cm	
Carriage speed = 2.5 cm/s			
Test No.	Pre-cuts	Experimental Results, N	
1	none	H = 174 ± 50 (274) V = 148 ± 39 (226)	
2 ¹	radial		H _b = 67 V _b = 55
3 ¹	radial		H _b = 80 V _b = 73
4 ¹	circumferential		H _b = 127 V _b = 105
5 ¹	none	H _b = 124 V _b = 114	

¹Carriage speed 0.3 cm/s, breaking loads only

TABLE 5. Measured horizontal and vertical loads on 45-deg upward-breaking inclined plane structure - ice sheet No. 37

Structure characteristics		Ice characteristics	
B = 100 cm		$\sigma_f = 113 \text{ kPa} +$	
Z = 18 cm		$\sigma_f = 69 \text{ kPa} +$	
$\mu = 0.02$		E = 240 MPa	
		h = 4.8 cm	
		l = 70 cm	
Carriage speed = 2.5 cm/s			
Test No.	Pre-cuts	Experimental Results, N	
1	none	H = 310 $\pm$ 113 (536) V = 305 $\pm$ 97 (499)	
2 ¹	radial	H _b = 147 V _b = 134	
3 ¹	circumferential	H _b = 480 V _b = 380	
4 ¹	none	H _b = 407 V _b = 347	
5 ¹	radial	H _b = 150 V _b = 127	
6 ¹	radial	H _b = 140 V _b = 120	

¹Carriage speed 1.1 cm/s, breaking loads only

FIG. 8 Comparison of measured and predicted loads for ice failing against a 45-deg upward-breaking inclined plane

does not lend itself to a closed-form solution, a finite element approach would be more suitable. Similarly, failure load has been calculated in terms of flexural strength. An alternative approach would be to employ fracture mechanics to determine the load necessary to create a stress in the ice capable of propagating a crack.

Another area of uncertainty is that of adding together the load components to obtain the maximum

total load. A simple sum of all the maximum values has been used, but this may not be realistic. What is required is a closer look at the temporal variations of the load components and the application of a probabilistic technique for summing them.

To provide an appreciation of a full-scale application, the analytical model was applied to a 2-m thick cold sea ice cover failing against a 25-m wide structure with a freeboard of 5 m. Friction coefficient of the ice on the structure was taken as 0.3, flexural strength as 750 kPa, and elastic modulus as 3 GPa. The horizontal components of breaking, rotating, and sliding forces were 3, 7 and 13 MN, respectively. The structure would react to this load as to a pressure of about 500 kPa, not an unreasonable value. A caution has to be expressed here for prototype structures, since the broken ice pieces have to be dealt with after they go over the top of the structure. This can in some cases be a greater problem than simply resisting the direct global ice loads.

CONCLUSION

A series of tests has been carried out to help to define and verify an analytical model for predicting ice loads on an inclined structure. The model applies to intermediate and wider structures (structure width at least five times ice thickness) and does not deal with the problem of broken ice once it overtops the structure.

1. Failure of the ice cover is controlled by the formation of radial cracks in front of the structure.

TABLE 6. Predicted components and maximum total horizontal load on 45-deg upward-breaking inclined plane structure for condition of model basin tests

Ice Sheet, Test No.	Breaking		Rotation	Sliding	Total	Experimental (95%)
	Radial	Circumferential				
33-1	143	(73) ¹	115	134	392	283
33-2	0	0	81 ²	111 ²	192	172
33-4	143	0	39 ²	81 ²	263	218
33-5	0	73	115	134	322	228
34-1	143	(73)	131	143	417	273
34-2	143	0	38 ²	79 ²	260	252
34-3	127	(66)	120	137	384	294
34-4	127	0	39 ²	81 ²	247	204
34-5	89	(46)	111	131	331	302
34-6	89	0	39 ²	81 ²	209	199
35-1	137	(77)	117	228	482	363
35-2	151	0	54 ²	192 ²	397	397
35-4	0	84	117	228	429	233
36-1	92	(37)	83	155	330	274
36-2	0	39	85 ²	0	124	67
36-3	0	38	84 ²	0	122	80
36-4	92	0	83	0	175	127
36-5	92	(37)	83	0	175	124
37-1	186	(118)	172	238	596	536
37-2	0	118	172		290	147
37-3	186	0	172 ²		358	480
37-4	186	(118)	172		358	407
37-5	0	118	172		290	150
37-6	0	118	172		290	140

¹Parenthesis indicate that circumferential load was not used in determining total load.

²Precut size of pieces used to calculate rotational and sliding forces.

2. Subsequent formation of a circumferential crack at a distance from the structure of three quarters of the ice sheet characteristic length was indicated by the analytical model and verified by observation.
3. The three components of load acting on an inclined structure include breaking, rotation, and sliding.
4. An analytical model based on the three identified load components predicts total loads about 20% greater than those measured. This is to be expected, given the conservative assumptions in the development of the model.
5. The analytical model has application for predicting full-scale loads and for carrying out parametric studies to establish the influence of structure characteristics and ice properties on total ice load.

ACKNOWLEDGEMENT

The technical assistance of R. Bowen in the model tests is gratefully acknowledged. This paper is a contribution from both the Division of Building Research and the Division of Mechanical Engineering, National Research Council of Canada.

REFERENCES

1. Tryde, P., Intermittent ice forces acting on inclined wedges, U.S. Army, Cold Regions Research and Engineering Laboratory, Hanover, N.H., Report 77-26, 1977.
2. Sorensen, C., Interaction between floating ice sheets and sloping structures, Institute of Hydrodynamics and Hydraulic Engineering, Technical University of Denmark, Series Paper No. 19, 1978.
3. Frederking, R.M.W., Dynamic ice forces on an inclined structure, *In* Physics and Mech. of Ice (P. Tryde, ed.), Springer-Verlag, Berlin, 1980, pp. 104-116.
4. Croasdale, K.R., Ice forces on fixed, rigid structures, U.S. Army, Cold Regions Research and Engineering Laboratory, Hanover, N.H., Spec. Rept. 80-26, 1980, pp. 34-106.
5. Ralston, T.D., Ice force design considerations for conical off-shore structures, Proceedings 4th International Conference on Port and Ocean Engineering under Arctic Conditions, St. John's, Newfoundland, 1978, pp. 741-752.
6. Abdelnour, R., Model tests of an icebreaking inclined plane, Arctec Canada Limited, Ottawa, Report No. FR 289C-2, 1979.
7. Timco, G.W., Model tests of ice forces on a wide inclined structure, Proc. IAHR Symposium on Ice, Vol. 2, Hamburg, 1984, pp. 87-96.
8. Nevel, D.E., A semi-infinite plate on an elastic foundation, U.S. Army, Cold Regions Research and Engineering Laboratory, Hanover, N.H. Res. Rept. 136, 1965.
9. Lavrov, V.V., Deformation and strength of ice, (Translated from Russian), Israel Program for Scientific Translation Ltd., U.S. Department of Commerce, National Technical Information Service, Springfield, Va., TT70-50130, 1971, 164 p.
10. Nevel, D.E., The ultimate failure of a floating ice sheet, IAHR Symposium on Ice and its Action on Hydraulic Structures, Leningrad, 1972, pp. 17-22.

reprinted from

**Proceedings of the Fourth International Offshore Mechanics and
Arctic Engineering Symposium – Volume II**

Editor: Jin S. Chung
(Book No. I0185B)

published by

THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
345 East 47th Street, New York, N.Y. 10017
Printed in U.S.A.

This paper, while being distributed in reprint form by the Division of Building Research, remains the copyright of the original publisher. It should not be reproduced in whole or in part without the permission of the publisher.

A list of all publications available from the Division may be obtained by writing to the Publications Section, Division of Building Research, National Research Council of Canada, Ottawa, Ontario, K1A 0R6.

Ce document est distribué sous forme de tiré-à-part par la Division des recherches en bâtiment. Les droits de reproduction sont toutefois la propriété de l'éditeur original. Ce document ne peut être reproduit en totalité ou en partie sans le consentement de l'éditeur.

Une liste des publications de la Division peut être obtenue en écrivant à la Section des publications, Division des recherches en bâtiment, Conseil national de recherches Canada, Ottawa, Ontario, K1A 0R6.