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Numerical Study of Laminar Flow and Mass Transfer for In-Line Spacer-Filled Passages

Performance calculations for laminar fluid flow and mass transfer are presented for a passage containing cylindrical spacers configured in an inline-square arrangement, typical of those employed in the process industries. Numerical calculations are performed for fully-developed flow, based on stream-wise periodic conditions for a unit cell and compared with those obtained for developing regime in a row of ten such units. The method is validated for an empty passage, i.e., a plane duct. Results are presented for the normalized mass transfer coefficient and driving force, as a function of mean flow Reynolds number, and also the wall mass flux, or blowing parameter. Both constant and variable wall velocities were considered, the latter being typical of those found in many practical membrane modules. [DOI: 10.1115/1.4007651]

Keywords: computational fluid dynamics, periodic boundary conditions, mass transfer, membrane technology

Introduction

Membranes provide solutions for reverse-osmosis, ultra-filtration, and nanofiltration applications. Membrane modules may be planar, tubular, or spiral-wound. Spacers are employed in membrane modules to provide structural integrity and promote mixing. This mixing increases the overall mass transfer coefficient, or conductance, and thereby improves performance. A module typically involves hundreds or thousands of spacer elements, within which the flow and concentration can be shown to be fully-developed. Better design and construction of membrane modules reduces cost and increases efficiency and reliability of filtration systems. Understanding and optimizing fluid flow and mass transfer in such modules can be achieved by means of computational fluid dynamics (CFD).

In a conventional membrane module, the feed-retentate mixture flows between two parallel membranes separated by a distance, h , through which the permeate or transferred substance passes, but which does not admit the retentate or considered substance (or only admits a certain fraction). Figure 1 shows a single-leaf, spiral wound membrane module, together with the details of the spacer geometry. The space between the two membrane walls, height $h = 2.01$ mm, is occupied by a mesh of equally-sized circular solid filaments, diameter $d = 1.03$ mm, and pitch $l = 2.17$ mm, intersecting at 90 deg and oriented normally to the main flow. The so-called Conwed spacer geometry was previously considered by Da Costa et al. [1–3] who performed flow field calculations and gathered experimental data on overall pressure drop, and also by Karode and Kumar [4] who conducted a 3D CFD analysis of the fluid flow. Studies on rectangular and curvilinear channels have also been reported [5]. Although some authors have established correlations with dimensionless numbers [6–8], a systematical rationalization of the mass transfer problem in this area is still lacking. This paper presents the fundamental theory and results for combined fluid flow and mass transfer in the Conwed spacer geometry under both developing and fully-developed conditions. Mass transfer in plane ducts for parabolic flow (no recirculation) were

considered by Beale [9], who also described a theoretical methodology for periodic heat/mass transfer boundary conditions [10,11]. Darcovich et al. [12] subsequently considered mass transfer in a plane duct bounded by a porous membrane with stream-wise periodic boundary conditions. In this study, the methodology is extended to include the presence of convective mass transfer at the boundaries for more complex situations. The mass transfer problem is systematically rationalized in terms of appropriate dimensionless numbers to reduce the functional dependence on a number of variables in spacer-filled channels.

Mathematical Considerations

The transport equations were solved for steady incompressible laminar fluid flow and binary mass transfer with constant properties

Fig. 1 Schematic of spiral-wound membrane module showing details of Conwed spacer geometry

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$$\text{div}(\rho \mathbf{u} \mathbf{u}) = -\text{grad } p + \text{div}(\mu \text{grad } \mathbf{u}) \quad (2)$$

$$\text{div}(\rho \mathbf{u} \mathbf{y}) = \text{div}(\Gamma \text{grad } y) \quad (3)$$

Values of transport properties were selected as follows; $\rho = 1.0163 \times 10^3 \text{ kg/m}^3$, $\nu = 2.427 \times 10^{-3} \text{ m}^2/\text{s}$, $\text{Sc} = 1$. In the finite-volume method [13], sets of linear algebraic equations of the form

$$\sum a_{NB}(\phi_{NB} - \phi_P) + S = 0 \quad (4)$$

are solved. The source term, S , is typically linearized with respect to the in-cell value, ϕ_P , according to

$$S = C(\phi_w - \phi_P) \quad (5)$$

where C is a source-term coefficient, ϕ_w is the value of ϕ at the wall. The notation follows that of Spalding [14,15]. Patankar [13] employs the equivalent form, $S = S_C + S_P \phi_P$, with $S_P = -C$ and $S_C = C \phi_w$.

The “Ohm’s law” of convective mass transfer [16,17] relates the local mass flux to the local driving force as

$$\dot{m}'' = gB \quad (6)$$

The local mass transfer coefficient or conductance, g , is evaluated from the normal flux on surfaces 1 and 2, Fig. 1(b) (and denoted below by g_1 and g_2) according to

$$g = \frac{-\Gamma \partial y / \partial n|_w}{y_w(x, z) - y_0(x)} \quad (7)$$

The local mass transfer driving force, B , also known as the Spalding number, is defined as

$$B(x, z) = \frac{y_0(x) - y_w(x, z)}{y_w(x, z) - y_t} \quad (8)$$

where y_t is the transferred substance-state [16,17]. The reference bulk value used to enumerate local values of B in Eq. (8) is the local bulk mass fraction

$$y_0(x) = \frac{1}{\rho u_0 h l} \int_0^h \int_0^l \rho u y dy dz \quad (9)$$

In this equation (only) dy refers to an element of displacement, and here and elsewhere y refers to mass fraction, as is the usual notation. The wall boundary condition for the transferred substance, or permeate, may be written as

$$S = \dot{m}'' A [y_t - y_w(x, z)] \quad (10)$$

The wall-value and source-term coefficient are computed as harmonic averages

$$y_w = \frac{\Gamma / \delta}{\Gamma / \delta + \dot{m}''} y_P + \frac{\dot{m}''}{\Gamma / \delta + \dot{m}''} y_t \quad (11)$$

$$\frac{1}{C} = \frac{\delta}{\Gamma A} + \frac{1}{\dot{m}'' A} \quad (12)$$

where y_P refers to the nodal value of y [13] at the computational cell immediately adjacent to the wall. Equation (12) expresses the fact that the resistances are in series. For very fine meshes, $y_w \approx y_P$, and $C \approx \dot{m}'' A$.

The flow is characterized by the main flow Reynolds number, based on mean interstitial velocity

$$\text{Re}_0 = \frac{\rho u_0 d_h}{\mu} \quad (13)$$

where $d_h = 4\varepsilon/[2/h + 4(1 - \varepsilon)/l] = 0.997 \text{ mm}$ is the hydraulic diameter and $\varepsilon = 1 - \pi d^2/2lh = 0.618$ is the fluid volume-fraction, or porosity. (Only for $h \geq 2d$, which is strictly not true here. However, it is accurate to better than 1%.) The superficial velocity is just, $U_0 = \varepsilon u_0$. A wall Reynolds number is similarly defined, $\text{Re}_w = \rho v_w d_h / \mu$. For fully-developed flow it is assumed that the upstream boundary value is

$$y(0) = c_1 y(l) + c_2 \quad (14)$$

$$c_1 = \frac{y_0(0) - y_w(0, y, z)}{y_0(l) - y_w(l, y, z)} \quad (15)$$

$$c_2 = y_0(0) - c_1 y_0(l) \quad (16)$$

For elliptic problems, downstream, $y(l) = c'_1 y(0) + c'_2$, where $c'_1 = 1/c_1$ and $c'_2 = -c_2/c_1$ [10,11]. The latter were not required, owing to the choice of the grid boundaries at locations where the flow was “locally-parabolic,” i.e., no local recirculation [13]. One advantage over previous methods [18,19] is that primitive variables are employed, and therefore a linear (Robin) wall boundary condition, Eq. (5), may readily be accommodated [20]. The upstream wall values must be computed from the downstream values

$$y_w(0, z) = \frac{y_0(0) + B(l, z) y_t}{1 + B(l, z)} \quad (17)$$

The upstream bulk value, $y_0(0)$, is prescribed by the user whereas the downstream value $y_0(l)$ is computed. Periodicity is defined by the driving force being cyclic, namely

$$B(l, z) = B(0, z) \quad (18)$$

It was shown in Ref. [20], which Eq. (18) generated results consistent with known values for heat transfer in a plane duct over a wide range of wall boundary conditions. For very low mass transfer rates, the problem is analogous to heat transfer with constant wall flux (Neumann problem), and no continuity source terms are required. However for finite-rate wall mass transfer, the inlet/exit velocity fields differ in magnitude due to injection/suction. Under those circumstances the stream-wise velocity may also presumed to be periodic with

$$u(0, y, z) = cu(l, y, z) \quad (19)$$

i.e., a similarity profile is assumed. By analogy with Eq. (10) a wall momentum-source in the cross-wise velocity field, $S = \dot{m}'' A (v_t - v_w)$ must be introduced, with periodic terms at the inlet/outlet.

The mean drag coefficient is defined in terms of the interstitial velocity as

$$f = \frac{d_h}{l} \frac{\Delta p}{\frac{1}{2} \rho u^2} \quad (20)$$

The overall Sherwood number is evaluated from the mean driving force, $\bar{B}$, at both the top and bottom surfaces as

$$\bar{\text{Sh}} = \text{Pe}_w / \bar{B} \quad (21)$$

where $\text{Pe}_w = \text{Re}_w \text{Sc} = \dot{m}'' d_h / \Gamma$. $\bar{B}$ is just an arithmetic average of B , determined from the globally-averaged average wall mass fractions

$$\bar{y}_w = \frac{1}{l^2} \int_0^l \int_0^l y_w dx dz \quad (22)$$

computed at both the top and bottom surfaces, and the globally-averaged bulk value,

$$\bar{y}_0 = \frac{1}{l} \int_0^l y_0 dx \quad (23)$$

The Stanton number for mass transfer is computed as

$$\overline{St} = \frac{\overline{Sh}}{Pe_w} = \frac{1}{\bar{B}} \quad (24)$$

Following previous work, a mean blowing parameter is defined as

$$\bar{b} = \frac{\bar{m}''}{g^*} \quad (25)$$

where $g^* = g$ in the $\lim \bar{m}'' \rightarrow 0$, and the overall conductance, $\bar{g}^* = \overline{Sh}^* \Gamma / d_h$. For $\bar{b} \ll 1$, mass transfer is sufficiently small that continuity and momentum effects due to injection/suction at the wall may be ignored.

For the developing situation, a total of ten unit cells in the streamwise direction were considered with the inlet and exit zones, as shown in Fig. 2. Owing to the symmetry of the problem, the extent of the geometric domain was $(12 \times l) \times h \times l/2$. A computational grid of $(12 \times 32) \times 80 \times 32$ was concentrated towards $y = \pm h/2$ by means of a geometric progression using a Cartesian cut-cell approach [21,22] for the solid-fluid interfaces. Similarly, for the periodic problem a $32 \times 80 \times 32 = 8,190$ cell grid of size $l \times h \times l/2$, was constructed. The front and back boundaries are treated as symmetry planes, whereas the top and bottom boundaries are prescribed as walls. The grid is aligned with the main flow direction, thus minimizing numerical diffusion. The CFD code PHOENICS version 2008 [23] was used to perform the majority of the calculations.

Tests were conducted to validate the numerical procedure, by removing the cylindrical obstacles, and associated cut-cells, from the domain. Values of $f^*/4 = 0.2399$ and $Sh^* = 8.228$ for the Neumann problem were observed with $v_w = 10^{-10}$ and $y_i = 10^{+10}$ [15], compared with $f^*/4 = 0.24$ (−0.04%) and $Sh^* = 8.235$ (−0.08%) from Kays et al. [24] at $Re_0 = 100$. Numerical benchmarks for the equivalent heat transfer problem may be found in Ref. [20]. These differ from the finite-rate mass transfer problem here, as follows. Figure 3 shows values of g/g^* , as a function of the blowing parameter, b . The reader will note that for $b > 0$ (injection), $g/g^* \rightarrow 0$ as $b \rightarrow \infty$, and for $b < 0$ (suction) $g/g^* \rightarrow \infty$ as $b \rightarrow -\infty$. This is in contrast to heat transfer in the absence of injection/suction where values of g/g^* would range from 1 down to only 0.916, i.e., $Nu = 7.541$ [24], corresponding to a Dirichlet

Fig. 2 Grid details (a) single unit and (b) 10 unit cells

Fig. 3 Benchmark studies for a plane duct

condition (solid red line in Fig. 3). The reason for this is that for the latter problem, the v -velocity is negligibly small, whereas for the former the v -velocity exhibits a parabolic profile the magnitude of which increases with b , with a maximum at the wall [25]. The presence of this lateral convective flux leads to much larger variations in the mass transfer coefficient than for the classical heat transfer problem, where only diffusion is significant in the lateral direction. Since there are no wall mass sources in the continuity and momentum equations for the heat transfer problem, the mass flux, $\bar{m}''$ or b in Fig. 3, is “virtual” (i.e., in the scalar equation only) ranging from zero (Neumann) to a very large number (Dirichlet), see Eq. (10) and Refs. [14,15]. While for many low mass-transfer situations, the Neumann solution is adequate, it is important to appreciate that for high mass transfer rates, the cross-wise velocity is important, and cannot be considered without also taking into account continuity effects, due to pressure-coupling effects being present. The close agreement between the numerical g/g^* profile and the analytical solution, $g/g^* = b/(\exp b - 1)$, reinforces the suggestion [16,17] that the 1D solution may be used to estimate g from g^* , at least for simple ducts.

Grid Independence and Choice of Numerical Scheme. The numerical accuracy of the calculations was investigated by repeatedly doubling the mesh in each direction. With a $16 \times 40 \times 16$, $32 \times 80 \times 32$, and $64 \times 160 \times 64$, the percent changes in f^* compared to the finest grid are −3.1% and −0.3%, whereas those for Sh_1^* are −1.6% and −0.6%, and for Sh_2^* −8.3% and −2.1%. A hybrid-differencing scheme (HDS) [26] was employed as the default convection-diffusion scheme, to facilitate coding the periodic boundary conditions across the jump boundaries. Numerical diffusion effects were further investigated by considering a quadratic upwind for convection kinematics (QUICK) scheme, Leonard [27], and also a cubic upwind scheme (CUS), for the ten-row developing problem. It was found that for friction factor, QUICK is 0.25% higher than HDS, whereas CUS is only 0.03% higher. Similar values of Sh_1^* and Sh_2^* are +3.21% and −3.54% (QUICK) and −3.04% and −3.12% (CUS) compared HDS, suggesting that with the grid oriented in the main flow direction, little is gained with higher-order schemes. Also, a criticism of higher schemes, is that they suffer from unboundedness, especially in the presence of discontinuities [28], as occur here.

Convergence was obtained readily for both periodic, and developing schemes at low Reynolds numbers. Figure 4 shows spot values, normalized with respect to the minimum and maximum observed values, for the periodic scheme. It can be seen there are some initial oscillations in spot values as the periodic boundary values stabilize. The magnitude of these oscillations will vary depending on the level of relaxation, and also, somewhat arbitrarily, on the minimum and maximum values of the spot value, which typically occur in the first few iterations. It can be seen that values are stable after about 150 iterations. Figure 5 is a plot of

Fig. 4 Normalized spot values, periodic boundaries

Fig. 5 Residual plot, periodic boundaries

the residuals, i.e., the sum of the square of values on the left-hand side of Eq. (4). For pressure-correction, they are mass imbalances. It can be seen that convergence is satisfactory with the decrease in the residuals for u , v , w preceding that for p and y . While a small performance penalty, in terms of convergence time, is noted for the periodic methodology; this is offset by the large reduction in computational domains required to obtain a solution to the problem.

Results

Low Constant-Rate Mass Flux. Calculations were performed for the spacer-filled geometry under a variety of boundary conditions. Figure 6 shows comparisons of calculations performed for the fully-developed periodic problem (horizontal lines) with those for the 10-unit-cell entry length case (symbols represent values at each unit-cell). It can be seen that developing values of f^* , and Sh_1^* , Sh_2^* , defined on the two surfaces illustrated in Fig. 1, asymptotically tend towards fully-developed values, with convergence occurring further upstream at lower Reynolds numbers. Sh_1^* values are generally larger than Sh_2^* values, the latter approaching fully-developed values more rapidly, owing to the presence of the continuously repeating periodic disturbances of the flow by the transverse obstacles.

Figure 7 shows streamlines corresponding to the flow field at a Reynolds number, $Re_0 = 80$. It can be seen that a wake vortex has developed behind the transverse cylinder, surface 2, Fig. 1(b), and that this leads to a stream-wise horseshoe-shaped “passage” vortex at the front of the subsequent downstream cylinder, as the velocity is larger in the free stream than near the longitudinal cylinder, surface 1, Fig. 1(b).

Figure 8 shows values of f^* , St_1^* , St_2^* , for $Sc = 1$, at various Reynolds numbers. Values of the friction factor, f^* , are similar in magnitude to those for an empty channel (plane duct), however, because the fluid volume-fraction, $\varepsilon = 0.618$, the total mass flow is reduced, i.e., drag is effectively increased by the presence of the cylinders. It can be seen that for $Re_0 > 100$, f^* -values for

(a)

(b)

Fig. 6 Friction and mass transfer factors for multirow developing and fully-developed periodic flow. $Re_0 = 20$ (upper) and $Re_0 = 60$ (lower).

Fig. 7 Flow visualization for $Re_0 = 80$

Fig. 8 Friction and mass transfers factor, fully-developed periodic flow, constant v_w

presumed steady flow, start to deviate from results obtained for flow-field calculations based on unsteady direct numerical simulations. The latter were obtained using a different code, OpenFOAM (Open Field Operation and Manipulation) [29,30], on a 500,000 node unstructured mesh with 50,000 time steps. The agreement between the numerical calculations at low Re_0 is encouraging. Also shown are the measured experimental f^* -data of Da Costa et al. [1–3]. The latter are for higher Reynolds numbers than were considered in this study. The present results are also consistent with earlier results [31] obtained using other CFD codes, where it became apparent that it is difficult to obtain converged steady-solutions at $Re_0 > 100$. It should be clear that while the trend in f^* continues with increasing Re_0 , it is necessary to use an unsteady scheme to properly capture this flow regime. Attention in this paper is however confined to the low- Re_0 steady regime.

Figure 9 illustrates local retentate (solute) wall mass fraction profiles for suction (b negative), with an inlet retentate mass fraction of $y_0(0) = 0.03$. A constant wall velocity $v_w = -5 \times 10^{-4}$ m/s, was imposed at both walls. Figure 10 shows the corresponding values of local Sherwood number, Sh_1^* and Sh_2^* . The mean wall retentate mass fractions are $\bar{y}_1 = 0.0392$ and $\bar{y}_2 = 0.0492$, with $\bar{Sh}_1^* = 1.82$ and $\bar{Sh}_2^* = 0.63$. Thus both Sherwood numbers are substantially less than for an empty duct, indicating a performance reduction associated with this spacer configuration. It can be seen that retentate wall mass fractions are a maximum in the stagnation zones where the cylindrical spacers touch the walls, and a minimum in regions where stream-wise convective transport is high. Mass fractions are higher on surface 1 than surface 2. Surface 1 is exposed to a shear layer due to the locally accelerated flow which increases the concentration gradient. The presence of the longitudinal cylinders creates a u -velocity profile which is parabolic-shaped in the z -direction near surface 1 (unlike a plane duct where it is constant in the span-

Fig. 9 Mass fraction of retentate, y_w , at surfaces 1 and 2, $Re_0 = 80$, constant v_w

Fig. 10 Local values of Sh at surfaces 1 and 2, $Re_0 = 80$, constant v_w

wise direction), resulting in a maximum local Sh_1^* in the centre-plane and negligibly small value for Sh_1^* at the cylinder wall(s). For mass transfer from surface 2, the lateral cylinders create a shadow effect, and hence $\bar{Sh}_2^*$ is smaller than $\bar{Sh}_1^*$. At both surfaces, local values are negligibly small directly under the cylinders, where there is little or no mass transfer. Since the magnitude of the driving force, B , is proportional to Δy , it follows B is a maximum and g a minimum in regions of low concentration (for suction). Therefore, Sh_1 is a maximum in the free shear layer, where the velocity and concentration gradients are large. Sh_2^* is influenced by the intertube vortex, which is relatively quiescent at low Reynolds numbers, therefore effecting more uniform mass fraction gradients towards the wall and resulting in lower overall Sh_2^* values. The definition of local Sh is ambiguous very near the cylinders, since the bulk fluid is physically separated from the wall region by the spacers.

Low Variable-Rate Mass Flux. The constant v_w boundary condition is an idealization of the situation which occurs in real membrane separation processes [32], where the wall velocity is coupled to the local concentration through the osmotic pressure [33]

$$v_w = L_p(\Delta p - \Delta \Pi) \quad (26)$$

where L_p is the membrane permeability, Δp is the pressure change across the membrane, and $\Delta \Pi$ is the osmotic pressure difference which is itself a nonlinear function of the concentration difference across the membrane. v_w is negative for suction, positive for injection. With reference to Fig. 11, Pharoah [31] proposed a linear approximation to the osmotic pressure profile for dilute NaCl brine, when compared with the data of Perry et al. [34]

Fig. 11 Osmotic pressure as a function of retentate mass fraction for NaCl-brine, adapted from Pharoah [31]

$$v_w = v_0 + Ay_w \quad (27)$$

where y_w refers to the retentate, and $v_0 = L_P \Delta p$, i.e., it is presumed that local variations in the hydraulic pressure are small compared to the pressure difference across the membrane. A regression analysis suggests $A = 1.214 \times 10^{-4}$ m/s. The source-term coefficient, C , in Eq. (5) is now essentially a linear function of y_w , assuming the convective flux dominates the diffusion flux in Eq. (12), and thus the source term, S , displays a quadratic dependence on y_w .

Tests conducted on the 10-cell domain compared favorably with the 1-cell periodic solution, as before. Figure 12 shows wall mass flux, $\dot{m}''$, at surfaces 1 and 2. It can be seen that near the cylinders where the solute concentrations are relatively high, the magnitude of the wall velocity is reduced due to the lower osmotic pressure difference. For the very dilute mixture and associated small mass flux considered here, this effect is second-order and the impact upon the wall mass fractions and Sherwood numbers is negligible; However for larger values, the changes in the concentration will significantly affect the local wall velocity, hence mass fractions and local Sherwood numbers will become problem-specific. A detailed parametric study of these effects is considered beyond the scope of this paper. Decreases in mass fraction of the solute can also lead to substantial changes in the magnitude of the transpiration velocity over the length of the entire membrane assembly. Although the variation over a single periodic element is typically small, the difference between the inlet and the outlet of an entire module will often result in a significantly different blowing parameter.

The linear formulation for v_w , Eq. (27), is considered sufficient to demonstrate the functionality of the present method. It is, of course, straightforward to adopt higher-order regression polynomials when higher-concentration mixtures are considered. The assumption that the retentate mass fraction outside the membrane wall is negligibly small (i.e., $y_r = 1$ for the transferred substance) may also be relaxed.

High Constant-Rate Mass Transfer. For low Re_w , the velocity profile is unaffected by injection or suction at the wall. Under such circumstances it is not necessary to account for continuity or momentum effects. The converse is true for high mass transfer rates where it is necessary to modify the stream-wise periodic algorithm to take these effects into consideration. At high Re_w , the streamwise velocity field can no longer be considered cyclic, but instead periodic, according to Eq. (19), and additional sinks of mass and momentum at the walls, and equal and opposite sources (or vice versa) across the periodic boundary, must be prescribed. The treatment is essentially the same as was already described for scalar mass fraction in Eqs. (14)–(16).

Fig. 12 Mass flux, $\dot{m}''$, of permeate at surfaces 1 and 2, $Re_0 = 80$, $v_w = v_0 + Ay_w$

Figure 13 shows values of g/g^* for high mass flux. It can be seen that for $b \ll 0$ (strong suction), mass transfer enhancement is significantly higher than for the plane duct case, shown in Fig. 3, though such high mass transfer rates are seldom achieved in most practical membrane applications. The results show that the concentration gradients at both walls are higher than for an empty duct, due to substantial mixing occurring in the central zone, and that this is further enhanced by strong suction. Of course at sufficiently high lateral mass flux rates the very notion of stream-wise periodicity becomes suspect due to cross-wise pressure gradients arising; however this is unlikely to be an issue for wall Reynolds numbers typical of membranes employed in the chemical-process industries.

Fig. 13 High rate mass transfer factors, constant v_w

Conclusions

The use of streamwise periodic boundary conditions allow for a detailed mass transfer analysis in spacer-filled membrane assemblies to be undertaken by consideration of a single unit-cell in place of multiple units. A large membrane module can thus be approximated by a small number of single-periodic ones, using the methodology described in this paper. The technique can be used for reverse osmosis (negligible cross-wise velocity) through to microfiltration (strong suction) units. The proposed scheme was validated with a 10-cell developing-flow assembly. The methodology was applied to constant wall velocity, and prescribed osmotic pressure boundary value problems. Convergence as measured by spot values and residuals, was readily achieved, however for $Re_0 > 100$, the solutions were found to be unsteady.

Maps of local Sherwood numbers which are useful in visualizing and improving mass transfer rates in present and future designs, were generated. Comparison of the fully-developed results for an empty duct, where $Sh^* = 8.23$, revealed that with the Conwed geometry local Sh^* ranged from a minimum of zero to a maximum of 4.5, showing the negative influence of the presence of cylindrical spacers upon mass transfer performance, for this compact inline spacer configuration. Increasing the longitudinal pitch may result in improvements. Local and overall Sh^* were substantially lower at the transverse spacer surface than the longitudinal cylinder surface due to the shading effect of the cross-wise spacers. Mass transfer in future designs could likely be increased by modifying the geometry, e.g., by increasing the spacer-pitch and/or changing the angle of attack of the fluid, for example to 45 deg.

Important considerations to be addressed in the future include; transient flow and mass transfer, and the impact of turbulence and unsteadiness upon drag and mass transfer. Correlating Sherwood number as a function of Schmidt number is important, as the latter can be quite large, e.g., for saline solutions. Local variations in Sc near the walls are also important. Comparisons of numerical results with those of reliable flow visualization experiments are ongoing and will be presented in future work.

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Nomenclature

A = area, m^2 , constant, m/s
 a = coefficient in finite-volume equations
 C = source-term coefficient in finite-volume equations
 d = diameter, m
 d_h = hydraulic diameter, m
 g = mass transfer coefficient, conductance, $kg/(m^2 \cdot s)$
 h = height, m
 l = unit-cell pitch, m
 L_p = hydraulic permeability, $m/(Pa \cdot s)$
 $\dot{m}''$ = mass flux, $kg/(m^2 \cdot s)$
 p = pressure, Pa
 S = source term
 u = mean interstitial velocity, m/s
 U = mean superficial velocity, m/s
 x = coordinate direction, m
 y = mass fraction, coordinate direction, m
 z = coordinate direction, m

Greek Symbols

δ = cell centre-to-wall distance, m
 Δ = change

ε = fluid volume fraction, porosity
 Γ = exchange coefficient, $kg/(m \cdot s)$
 ϕ = scalar variable
 Π = osmotic pressure, Pa
 ρ = density, kg/m^3
 μ = viscosity, $kg/(m \cdot s)$

Nondimensional Numbers

B = driving force
 b = blowing parameter
 f = friction coefficient
 Nu = Nusselt number
 Pe = Péclet number
 Re = Reynolds number
 Sc = Schmidt number
 Sh = Sherwood number
 St = Stanton number

Subscripts

0 = bulk
 t = transferred substance (permeate) state
 w = wall
 P = in-cell value

Superscripts

$-$ = mean, overall value
 $*$ = in the limit of negligible rate of mass transfer
 $''$ = per unit area
 $\cdot$ = per unit time

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