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by M.J. Ouellette

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SPECTRAL BANDING OR IRESCENCE WITH ANODIZED ALUMINUM REFLECTORS

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Abstract - Iridescence is a problem of increasing concern since the introduction of narrow band lamps and anodized luminaire surfaces. It appears as induced coloration of luminaire surfaces, much like rainbow patterns associated with thin oil or soap films on water. This paper discusses the causes of the effect and proposes an interim colorimetric method for measuring it.

INTRODUCTION

The lighting industry has greatly evolved since the introduction of the bare incandescent lamp. More than dispelling darkness, today's lighting creates form, mood, function and beauty. A wide range of products are now available for these purposes, including anodized aluminum reflectors and triphosphor fluorescent lamps.

Simulating clear mirrors, aluminum reflectors offer high reflectivity to both light and heat, providing high luminous efficacy and extended ballast and lamp life at lower operating temperature. They have been used successfully for years in fixture retrofitting and lighting energy conservation.

Triphosphor lamps are a new generation of fluorescent lamp featuring excellent color rendition. They are characterized by spectral power distributions dominated by three narrow peaks, nominally blue, green and red. Compact versions of triphosphor lamps are popular retrofit alternatives to incandescent lamps, including those in metal downlight fixtures. It is no surprise, then, that triphosphor lamps are frequently used with anodized aluminum reflectors. Perhaps surprisingly, though, discriminating clients occasionally express dissatisfaction with the appearance of faint, often ill-defined, colored bands on aluminum luminaire surfaces. Although the problem can occur with other types of narrow band lamps, it is perhaps most noted with triphosphor lamps.

This effect has become a matter of concern within the lighting industry and is commonly referred to as *spectral banding* or *iridescence*. Currently, there is no objective method for quantifying the extent of the effect. Until such a method is developed, it will remain difficult or impossible to establish standards and guidelines for product quality in regards to iridescence. Recognizing this need, Stannard [1] proposed an interesting computer and video based method for quantifying the effect. Its utility is currently limited, however, since measurements are in terms of the unknown spectral sensitivity of a conventional video camera and not in terms of human color vision. The method would have great potential once this fundamental problem is overcome. An obvious application would be in automated production line quality control.

This paper proposes an alternative method based on standard CIE [2] and ASTM [3] practices for the measurement of color in terms of the human visual system. It also provides, in modest detail, an analysis of the nature of the effect so that the problem at hand can be better managed and understood.

THE CAUSE OF IRESCENCE

Yahrous [4] reviewed the problem as it applies to the lighting industry. He proposed that iridescence is mainly attributed to the wavelength dependence of the index of refraction of anodized coatings, typically aluminum oxide monohydrate. Using Snell's Law of Refraction (1), he showed that mixtures of wavelengths emitted by fluorescent lamps separate as they refract at different angles, θ_2 ,

within the denser optical medium of anodized coatings (Fig. 1(a)).

$$\mu_2 \sin \theta_2 = \mu_1 \sin \theta_1 \quad (1)$$

Taking the index of refraction of air, μ_1 , as unity, and selecting the index of refraction, μ_2 , of the anodized coating in accordance with Fig. 2 for different wavelengths, Yahrous calculated that at an incidence angle, θ_1 , of 35° , blue light (wavelength, $\lambda=440$ nm) refracts at $\theta_2=20.138^\circ$ while red light ($\lambda=700$ nm) refracts at $\theta_2=20.278^\circ$, an angular difference of 0.140° , which he considers "optically significant." This would explain why iridescence is less noticeable with broad-band sources emitting energy in a wide range of different wavelengths each capable of combining and masking the weaker, separated wavelengths. Likewise, it would explain why iridescence is more noticeable with narrow-band lamps. No measurements were provided, however, to support the explanation [4].

Preliminary spectroradiometric measurements of an anodized aluminum reflector were collected at NRC/IRC in an attempt to verify Yahrous' explanation. The results indicate that the problem is not quite as simple as previously thought. The lower curve (c), in Fig. 3 represents the difference between (a) the spectral composition

Fig. 1. Reflection and refraction from a thin film, showing (a) different paths travelled by light originating from point S, and (b) phase difference between reflected and refracted waves, discussed later.

Fig. 2. Index of refraction of anodized aluminum oxide coatings (after Yahrous [4]).

of light reflected from an anodized aluminum surface[§] and (b) the spectral power distribution of the illuminant. Curves (a) and (b) are scaled on a criterion of equal luminance. Scaled in this manner, the two curves would coincide if the reflector were spectrally neutral. Differences between the two curves would depict the spectral influence of the reflector. From the periodic nature of (c), it is clear that discrete wavelengths are alternately enhanced and suppressed upon reflection - a phenomenon that cannot be explained by Snell's Law alone.

The periodic pattern in Fig. 3 is consistent with interference fringes associated with waves passing through a medium with dimensions of the same order of magnitude as the wavelength. This often applies to thin anodized coatings on luminaire surfaces designed for interior service (Table I). The phenomenon arises from interactions between reflected light from the upper surface of the coating (SAF) and light which has travelled the longer route (SABCG) through the anodized medium (Fig. 1(a)). Such interactions were investigated as early as

TABLE I
COATINGS FOR VARIOUS GRADES OF ANODIZED ALUMINUM, DERIVED FROM YAHROUS [1]

Grade	Typical Use	Coating Density mg/in ²	Coating Thickness mil (nm)	Perceived Iridescence
MI	Interior Commercial	5.0	0.10 (2540)	Moderate
SI	Interior Industrial	7.5	0.15 (3810)	Moderate
SE	Exterior Commercial	10	0.20 (5080)	Not Specified
M	Marine	13	0.26 (6604)	None

[§] The sample was a large-cell parabolic reflector of a 1'x4' troffer equipped with two warm white fluorescent lamps. Spectral measurements were obtained with a computer-driven PhotoResearch SpectraScan-PC spectroradiometer calibrated at the NRC, Division of Physics immediately prior to use.

Fig. 3. Spectral composition of an anodized aluminum reflector with warm white fluorescent illumination (a), spectral power distribution of a warm white fluorescent lamp (b), and the difference between the two (c), exaggerated by a factor of 5 for illustration purposes. Tick marks above and below the abscissa identify predicted wavelengths, $\lambda^{\uparrow}$ and $\lambda^{\downarrow}$, of maximum constructive and destructive interference.

the seventeenth century by Robert Boyle, Robert Hooke and Sir Isaac Newton. It was not until approximately 1801, however, that the phenomenon was correctly explained [5]. In consideration of Christian Huygen's wave theory for the motion of light, Thomas Young explained that as with waves from pebbles cast on water, intersecting light waves systematically reinforce and suppress each other at specific points related directly to their wavelengths.

To demonstrate, Young produced alternate light and dark bands from the overlapping waves of coherent light passing through adjacent narrow apertures. The bands were brightest at the intersection of wave crests, occurring at distances $\delta = 0, \lambda, 2\lambda, 3\lambda$, and so forth, from their source. They were dimmest at distances of $1/2\lambda, 3/2\lambda, 5/2\lambda$, and so forth. Thus, interference at any given distance, δ , depends on wavelength. Those wavelengths susceptible to maximum suppression and enhancement are designated, within this report, as $\lambda^{\downarrow}$ and $\lambda^{\uparrow}$ respectively, and are given as

$$\lambda^{\uparrow} = \delta / i, \quad (2)$$

$$\lambda^{\downarrow} = \delta / (i + 1/2), \quad (3)$$

for all positive integers, i .

Although anodized aluminum reflectors are typically not illuminated through narrow apertures, they are lighted through thin optical coatings (Table I) capable of inducing wave interference in much the same manner [5-8]. As in Young's experiment, there is a constant phase relationship between waves reflected from the front surface of the anodized coating and those refracted within it (Fig. 1(b)). The refracted component lags the reflected one by a fixed optical distance, δ , depending upon the index of refraction, μ_2 , the thickness, τ , of the anodized coating, and the wavelengths of incident radiation [5-8]. Equations (2) and (3) predict that only certain wavelengths will be maximally affected by any given phase lag. This preferential enhancement and suppression of wavelengths can induce color shifts in a manner consistent with the observed phenomenon of iridescence and the spectral differences in Fig. 3.

Fig. 4. As Figure 3, except for the pale purple iridescent region of an anodized aluminum reflector with SPX41 triphosphor illumination.

Under warm white fluorescent (WWF) illumination, the anodized luminaire surface represented in Fig. 3 appeared uniform and neutral in color. As with all broad band sources, WWF provides energy at many wavelengths. The greater the number of wavelengths, the greater the probability that systematic enhancement and suppression will cancel, resulting in little or no perceived color differences [6]. Triphosphor lamps, on the other hand, produce energy in considerably fewer wavelengths. Hence, changes in one or more peak wavelengths are more likely to produce changes in perceived color. In fact, under SPX41 triphosphor illumination, faint green and purple bands were evident on the same anodized surface as the one discussed above. Figure 4 shows spectroradiometric measurements of one of these bands. The predominant suppression in the green ($\lambda=542$ nm) region of the visible spectrum is consistent with its purplish appearance.

Figures (3) and (4) present compelling evidence that iridescence is most likely attributable to interference. The two phenomena are not necessarily synonymous, however. Thin film interference causes preferential enhancement and suppression of specific wavelengths. The changes in spectral composition may or may not be perceptible, since an infinite number of different spectral distributions are possible for any given perceived color [6]. Iridescence arises only when interference evokes a change in expected color appearance. The changes may be localized within a luminaire, where different portions of it appear in different colors (arbitrarily designated here as Type A iridescence). It might also appear as pale uniform tinting of the entire anodized surface, relative to that expected from the given illuminant (Type B iridescence). From casual experience, this type of iridescence is less objectionable than Type A, and, in many cases, would only be of concern when different luminaires appear differently colored within the same viewing area.

In principle, it is possible to predict the spectral distribution of light reflected from any anodized surface if the illuminant spectral power distribution, lighting geometry, and surface properties are known. For example, (2) and (3) can be used to determine $\lambda^\downarrow$ and $\lambda^\uparrow$, the wavelengths most susceptible to thin film interference. Here, δ is the optical path difference between the reflected (SAF) and refracted (SABCG) rays in Fig. 1(a), and is given by

$$\begin{aligned}\delta &= \mu_2(AB + BC) - \mu_1(AD) \\ &= 2\mu_2(AB) - \mu_1(AD).\end{aligned}$$

$$\begin{aligned}\text{Substituting } AD &= 2AE \sin \theta_1 \\ &= 2AE(\mu_2/\mu_1) \sin \theta_2 \quad (\text{from Snell's Law}),\end{aligned}$$

$$\text{and} \quad AE = AB \sin \theta_2,$$

$$\text{and} \quad AB = \tau / \cos \theta_2,$$

$$\begin{aligned}\text{gives } \delta &= 2\mu_2(\tau/\cos \theta_2)(1-\sin^2 \theta_2) \\ &= 2\mu_2(\tau/\cos \theta_2)(\cos^2 \theta_2) \\ &= 2\mu_2\tau \cos \theta_2.\end{aligned} \quad (4)$$

For example, if μ_1 is unity, θ_1 is 45° , coating thickness, τ , is 3900 nm, and μ_2 is given by Fig. 2, then one could iteratively solve (1), (4), (2) and (3) for various wavelengths and predict maximum destructive interference at $\lambda^\downarrow = 394, 407, 421, 436, 453, 470, 490, 510, 533, 558, 585, 616, 649, 687$ nm and maximum constructive interference at $\lambda^\uparrow = 387, 400, 414, 429, 444, 461, 480, 500, 522, 545, 571, 600, 632, 668$ nm. Interestingly, these points agree well with the positions of measured interference fringes in Fig. 3.

Of course, accuracy of prediction depends upon the accuracy of the input parameters, including τ and θ_1 . In practice, they are rarely constant. Reflector surfaces may be illuminated from many directions simultaneously. The surfaces may not be perfectly specular. They may be curved, dirty, and even textured to some extent. Coatings may be irregular in thickness and density, varying with such factors as current density, bath agitation, and position of electrodes used in the anodizing process [4]. With tolerance for these considerations, the close agreement between the predicted and measured positions of interference fringes in Fig. 3 lends strong support to the hypothesis that luminaire iridescence arises primarily from thin film interference.

Understanding the nature of iridescence, one can more effectively devise methods for measuring it, controlling it, eliminating it, and even taking advantage of it. For example, constructive interference can increase the effective amplitude of combined waves [5], with direct implications on lighting efficiency and ultimately on electrical energy requirements. Equations (1), (3), and (4) could be used to determine the optimum coating thickness for maximum reflectivity in the predominant wavelengths of a given illuminant.

MEASUREMENT OF IRIDESCENCE

As mentioned above, Type A iridescence arises from local color differences within a single luminaire, while Type B arises when different luminaires within a field of view appear in different colors. In both cases, the problem is one of unwanted color differences. It is therefore reasonable to measure the phenomenon using instrumentation designed specifically for the purpose of quantifying color in terms of the human visual system. Many colorimetric and spectroradiometric instruments are commercially available for this purpose. Most of them measure in the domain of the CIE 1931 (x,y) chromaticity space[‡] [2], one of the most popular spaces for the specification of color in engineering applications [3]. Within this space, two samples of equal lightness and equal chromaticity would appear, to the standard CIE 1931 photopic observer [2,6], as identical in color. Unfortunately, this space is nonuniform, meaning that equal changes in perceived color are not necessarily borne by equal changes in x and y -- a preponderant concern for quantifying color differences.

[‡] In brief, the x y coordinates represent relative weightings of three specific spectral distributions (nominally red, green, and blue) that together give the same color as the sample in question. The third coordinate, z, is implied from the other two, since $x+y+z$ equals unity by definition [6].

Fortunately, several approximately uniform chromaticity spaces already exist [2,3,9,10]. None are perfectly uniform [11], but most are suitable for specifying, within a reasonable degree of error, the small color differences associated with iridescence. Conveniently, some are simple numeric transforms of the familiar CIE 1931 space. Appendix I describes the steps for transforming CIE 1931 (x,y) coordinates into the CIE 1976 (L^* , u^* , v^*) approximately uniform color space [2]. It is commonly referred to as CIELUV, and is the one arbitrarily chosen for illustrating the concepts proposed here, although other chromaticity spaces could serve equally well.

The CIELUV chromaticity coordinates u^* and v^* are transforms of the CIE 1931 (x,y) coordinates. Because the CIELUV space is approximately uniform, it is possible to define chromaticity, u^* and v^* , relative to that of a reference sample (usually a nominally white light source). Distances between two points in this space denote approximate differences in color appearance.

Figure 5(a) and Table II show the relative chromaticity of a portion of an anodized aluminum reflector illuminated by a WWF lamp. The reference sample is conveniently the bare WWF lamp. By definition, its relative chromaticity lies at the origin of Fig. 5(a). The vector gives the magnitude, C_{uv}^* , and direction, h_{uv}^* , of the chromaticity shift between the bare lamp and the reflector sample. Although the vector shows a measurable chromaticity difference between the reflector and the bare lamp, all regions of the reflector's surface gave essentially the same vector. This uniformity of chromaticity over the reflector's entire surface is consistent with the apparent lack of iridescence for the reflector under WWF illumination.

Figure 5(b) and Table II show the relative chromaticity of two regions of the same reflector under SPX41 triphosphor illumination. The regions were the perceived centers of green and purple iridescent bands. The bare SPX41 lamp served as reference sample.

TABLE II
CIE 1931 AND CIELUV CHROMATICITY COORDINATES OF AN ANODIZED ALUMINUM REFLECTOR UNDER WWF AND SPX41 ILLUMINATION†.

	WWF		SPX41		
	Bare lamp	Reflector	Bare lamp	Reflector (green)	Reflector (purple)
x	0.4499	0.4476	0.3911 <u>0.3925</u> 0.3918	0.3934 <u>0.3934</u> 0.3934	0.3945 <u>0.3944</u> 0.3945
y	0.4109	0.4157	0.3944 <u>0.3923</u> 0.3934	0.3997 <u>0.4004</u> 0.4001	0.3905 <u>0.3912</u> 0.3909
Y	576.6	188.7	633.7 <u>614.7</u> 624.2	214.0 <u>216.0</u> 215.0	244.5 <u>245.8</u> 245.2
u^*	0.2559	0.2524	0.2259	0.2244	0.2286
v^*	0.5259	0.5274	0.5104	0.5133	0.5097
L^*	100	63.9	100	65.3	68.9
u^*	0.00	-2.94	0.00	-1.34	+2.41
v^*	0.00	+1.23	0.00	+2.53	-0.58
s_{uv}^*	0.00	0.0499	0.00	0.0438	.0360
C_{uv}^*	0.00	3.19	0.00	2.86	2.48
h_{uv}^*		157°		118°	346°

† Respective bare lamps serve as reference illuminants. Values below horizontal lines are averages of repeated measures, from which reported CIELUV parameters are derived. The spectroradiometer was repositioned between each measurement.

Fig. 5. CIELUV chromaticity shifts of (a) an anodized aluminum reflector under WWF illumination and (b) different regions of the same reflector under SPX41 illumination. The respective bare lamps served as reference illuminants.

Quite interestingly, the green and purple regions of the reflector surface gave different relative chromaticities.

The above measurements suggest a correlation between the magnitude of perceived iridescence and the range of relative chromaticity in a nominally uniform color space. It is therefore reasonable to formulate the working hypothesis that the greater the effect of iridescence, the greater the range of color shifts. If the points p and q in Fig. 5 represent the extremes of this range, then iridescence could be quantified in terms of the distance between them. Distances in planar Euclidean spaces may be characterized in a number of different manners [12]. The root sum of squared differences is the most common. In the space defined by Fig. 5, it is given as

$$II_0 = \sqrt{(\Delta u^*)^2 + (\Delta v^*)^2} \quad (5)$$

where Δu^* and Δv^* are the differences in u^* and v^* at the points p and q. The sample in Fig. 5 gave $II_0 = 0$ under WWF illumination and $II_0 = [(-1.34 - 2.41)^2 + (2.53 - (-0.58))^2]^{0.5} = 4.9$ under SPX41 illumination (Table II).

Equation 5 defines an operational Index of Iridescence, giving values near 0 for low iridescent materials, near 5 for moderately iridescent materials, and greater for highly iridescent ones. The equation is equally suitable for Type A and for Type B iridescence. The former requires measurements of small areas across a single sample. The latter requires measurement across different luminaires within an installation, or the same luminaire at different orientations. The points p and q could be found from manual spot measurements, or perhaps more accurately by automated scanning.

A number of practical issues should be considered before II_0 is implemented as a general measure of iridescence. For example, large measuring field sizes may decrease II_0 . Until this effect is explored further, only the smallest possible measuring field sizes should be used. Another consideration arises from the fact that the CIELUV space is three-dimensional. For convenience, II_0 ignores the third dimension, L^* , a correlate of lightness. The simplification is valid for sampled points* that do not differ greatly in luminance. This should not pose a serious problem for the measurement of most luminaire surfaces, provided that the fields of measurement do not include specular reflections.

There may well exist other metrics that are better correlated with the perceived magnitude of iridescence. These would become evident through additional colorimetric measurement and psychophysical experimentation. In addition, such work would assist in developing guidelines for implementing a practical method. Until then, II_0 is offered as an interim measure. It is believed that the concepts proposed here form a reasonable starting point.

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REFERENCES

- [1] S. Stannard, "Iridescence Measurement: a Computer Based Solution," in Proc. 1989 Annu. Conf. Illum. Eng. Soc. of N. Amer., pp. 525-534.
- [2] CIE Technical Committee TC-1.3, "Colorimetry," Commission Internationale de l'Eclairage, Vienna, Austria, Tech. Rep. CIE 15.2, 1986.
- [3] ASTM Committee E-12, ASTM Standards on Color and Appearance Measurement; Philadelphia, PA: Amer. Soc. for Testing and Materials, 1987.
- [4] T.G. Yahrous, "An Examination of the Spectral Banding Effect in Anodized Reflectors," in Proc. 1989 Annu. Conf. Illum. Eng. Soc. of N. Amer., pp. 511-524.
- [5] D. Halliday and R. Resnick, Fundamentals of Physics; New York, NY: John Wiley & Sons, 1970.
- [6] G. Wyszecki and W.S. Stiles, Color Science: Concepts and Methods. Quantitative Data and Formulae; Toronto, Ont.: John Wiley & Sons, 1982.

* The reference sample (e.g., the bare lamp) is not as susceptible to this luminance constraint. Errors in the absolute chromaticity of the reference would affect the relative chromaticity of all sampled points equally, and would cancel in (5).

- [7] R.W. Ditchburn, Light; New York, NY: Academic Press, 1976.
- [8] W.H.A. Fincham, Optics; London: Hatton Press, 1934.
- [9] G. A. Fry, "The Fry uniform chromaticity scale diagram," Color Res. & Appl., vol. 13, no. 3, pp. 153-165, June 1988.
- [10] T. Seim and A. Valberg, "Towards a uniform color space: a better formula to describe the Munsell and OSA color scales," Color Res. & Appl., vol. 11, no. 1, pp. 11-24, Spring 1986.
- [11] F. W. Billmeyer, "Quantifying color appearance visually and instrumentally," Color Res. & Appl., vol. 13, no. 3, pp. 140-145, June 1988.
- [12] B. Everitt, Cluster Analysis; New York, NY: Halsted Press, 1981.

APPENDIX I

Transformations for mapping CIE 1931 (x,y) into CIELUV approximately uniform color space. Values applicable to the reference illuminant are designated by the subscript $_n$. Details are available in the text and elsewhere [3-6].

X	$= (x/y)Y$	measured tristimulus values
Y	$= \text{luminance in cd/m}^2$	
Z	$= (z/y)Y$	
u'	$= (4X) / (X + 15Y + 3Z)$	approximately uniform chromaticity coordinates
v'	$= (9Y) / (X + 15Y + 3Z)$	
L^*	$= (116 \sqrt[3]{Y/Y_n}) - 16$	approximately uniform color space
u^*	$= (13L^*) (u' - u'_n)$	
v^*	$= (13L^*) (v' - v'_n)$	
C^*_{uv}	$= \sqrt{(u^*)^2 + (v^*)^2}$	CIELUV chroma
s^*_{uv}	$= C^*_{uv} / L^*$	CIELUV saturation
h^*_{uv}	$= \tan^{-1}(v^* / u^*)$	CIELUV hue