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EIGENMODE ANALYSIS OF THE INTERFERENCE PATTERNS IN REVERBERANT SOUND FIELDS

by

W. T. CHU

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Eigenmode analysis of the interference patterns in reverberant sound fields

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Results for the interference pattern in a reverberant sound field have been derived by means of an eigenmode analysis and compared with results obtained from the free-wave model. It is found that the incoherent or random nature of the sound field assumed in the free-wave model can only be achieved if the room is excited by random noise. For single frequency excitation an equivalent incoherent field can be approached by additional averages of results over many different source locations. The analytical solutions are supported by a computer study based on the eigenmode model. A detailed discussion about the correction factor due to boundary interferences in relation to sound power measurement is also given.

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INTRODUCTION

In a few recent articles,¹⁻⁵ evidence was presented in support of the view that the eigenmode and the free-wave models of the sound field in a reverberation chamber were consistent. For example, the spatial cross-correlation function, the spatial variance of the mean-square pressure $\bar{p}^2$, and the interference patterns at the boundaries of a rectangular reverberation chamber have been demonstrated to be derivable from either model.

This paper will show that the interference patterns near boundaries can also be obtained by both methods provided the assumptions involved are compatible. Thus it is necessary to differentiate between single frequency excitation and random noise excitation.

An important application of the prediction of boundary interference patterns is in sound power measurement. The inaccuracy in obtaining a true room-average mean-square pressure, when sampling is limited to the central portion of the room, can be compensated for by means of a boundary correction factor. In view of the current results, such a factor will be reexamined in greater detail.

I. EIGENMODE THEORY OF STEADY STATE SOUND

Let us consider a rectangular room of sides L_x, L_y, L_z and volume $V = L_x L_y L_z$. The sound field at the point r

$= (x, y, z)$ due to a simple source $Qe^{-i\omega t}$ at $r_0 = (x_0, y_0, z_0)$ can be written as^{6,7}

$$p = \frac{i\rho\omega Q}{V} e^{-i\omega t} \sum_N \frac{\psi_N(r)\psi_N(r_0)}{\Lambda_N(k^2 - k_N^2 + 2ik\delta_N)}, \quad (1)$$

where the origin of coordinates is at one corner of the chamber, N stands for the trio of numbers l, m, n and

$$\Lambda_N = 1/\epsilon_l \epsilon_m \epsilon_n, \epsilon_0 = 1 \text{ and } \epsilon_i = 2 \text{ for } i \neq 0,$$

$$\psi_N(x, y, z) = \cos k_x x \cos k_y y \cos k_z z,$$

$$k_x = \frac{l\pi}{L_x}, \quad k_y = \frac{m\pi}{L_y}, \quad k_z = \frac{n\pi}{L_z}, \quad k_N^2 = k_x^2 + k_y^2 + k_z^2,$$

$$\delta_N = \beta(\epsilon_l/L_x + \epsilon_m/L_y + \epsilon_n/L_z),$$

$k = \omega/c$ is the wavenumber, ρ is the density of the medium, and β is the specific normal admittance of the chamber surface assumed to be real and small. It is noted that only terms near $k_N = k$ contribute significantly because the damping term $2k\delta_N$ is usually small. In terms of complex notation, the mean-square pressure $\bar{p}^2$ can be obtained as $\frac{1}{2}(pp^*)$ where the bar denotes time averages and $*$ denotes complex conjugate. Using Eq. (1) to form the mean-square value gives

$$\bar{p}^2(r, r_0, \omega) = \frac{1}{2}(\rho\omega Q/V)^2 \times \sum_N \sum_M \frac{\psi_N(r)\psi_M(r)\psi_N(r_0)\psi_M(r_0)[(k^2 - k_N^2)(k^2 - k_M^2) + 4k^2\delta_N\delta_M]}{\Lambda_N\Lambda_M[(k^2 - k_N^2)^2 + (2k\delta_N)^2][(k^2 - k_M^2)^2 + (2k\delta_M)^2]}. \quad (2)$$

Equation (2) is the basic formula to be used in the following study of interference patterns near boundaries, i.e., the variation of $\bar{p}^2$ as a function of distance from boundaries. It is expected from Eq. (2) that answers might be different for three different types of excitation, namely, random noise excitation with single source position, pure-tone excitation with single source position, or averaging over many source positions. In

this paper the distinctions between these three cases will be properly considered.

II. RANDOM-NOISE EXCITATIONS

The normalized spatial cross-correlation function under narrow-band noise excitation was investigated previously⁵ by implicitly assuming that no two modes have the same frequency. Although this assumption

does not affect the normalized spatial cross-correlation function, nor the interference patterns to be studied here, it does change the absolute values of the mean-square pressure. In the present study, such an assumption is not used. Recognizing the fact that the empty rectangular room can be considered as a linear time-invariant system, Eq. (2) can be treated as the input-output relationship for a particular source-field points pair.⁸ Thus if the source emits random noise of equal energy (i.e., white noise), the mean-square pressure at the field point can be obtained by integrating Eq. (2) over frequency assuming Q^2 and β to be constants, i.e.,

$$\bar{p}^2(\mathbf{r}, \mathbf{r}_0) = \frac{1}{2} \left(\frac{\rho Q}{V} \right)^2 C^3 \sum_N \sum_M \frac{\psi_N(\mathbf{r}) \psi_M(\mathbf{r}) \psi_N(\mathbf{r}_0) \psi_M(\mathbf{r}_0)}{\Lambda_N \Lambda_M} \times \int_0^\infty \frac{k^2 [(k^2 - k_N^2)(k^2 - k_M^2) + 4k^2 \delta_N \delta_M] dk}{[(k^2 - k_N^2)^2 + (2k\delta_N)^2][(k^2 - k_M^2)^2 + (2k\delta_M)^2]} \quad (3)$$

A closed-form solution can be obtained for the integral in Eq. (3) only for the case with $N=M$ but not otherwise, i.e.,

$$\int_0^\infty \frac{k^2 dk}{(k^2 - k_N^2)^2 + (2k\delta_N)^2} = \frac{\pi}{4\delta_N} \quad (4)$$

In order to assess the importance of terms involving $N \neq M$, a numerical computation of the integral of Eq. (3) has been carried out for a number of values for k_N and k_M around a frequency of 1 kHz with room characteristics corresponding to NRC's big reverberation chamber. These are shown in Table I. Good comparison between results from the numerical integration and the closed form solution of Eq. (4) for cases involving $k_M = k_N$ serves as a check of the accuracy of our numerical method. As indicated by the results of Table I, the main contribution comes from terms with $N=M$ because the values of the integral are large for $N=M$, and the ψ 's for both the source and the field points are always positive, whereas, those for $N \neq M$ can be positive or negative and tend to average out to very small contributions. Thus in general Eq. (3) can be approximated as

$$\bar{p}^2(\mathbf{r}, \mathbf{r}_0) \approx \left(\frac{\rho Q}{V} \right)^2 \frac{\pi C^3}{8} \sum_N \frac{\psi_N^2(\mathbf{r}) \psi_N^2(\mathbf{r}_0)}{\Lambda_N^2 \delta_N} \quad (5)$$

Then if one is interested in narrow-band analysis, Eq. (5) can be rewritten as

$$\bar{p}_{\Delta\omega}^2(\mathbf{r}, \mathbf{r}_0) \approx \left(\frac{\rho Q}{V} \right)^2 \frac{\pi C^3}{8} \sum_N' \frac{\psi_N^2(\mathbf{r}) \psi_N^2(\mathbf{r}_0)}{\Lambda_N^2 \delta_N} \quad (6)$$

where $\sum'$ denotes summation over only those modes whose characteristic frequencies lie within the bandwidth of interest.

TABLE I. Tabulation of numerical results of the integral in Eq. (3) for a number of values for k_N and k_M around a frequency of 1 kHz. The value of the integral depends only on $|k_N - k_M|$.

$ k_N - k_M $	Value of integral
0.0	271.0
0.5	10.0
1.0	5.8
1.5	4.1
2.0	3.2
2.5	2.7
3.0	2.3
3.5	2.0
4.0	1.8
4.5	1.6
5.0	1.5

It is interesting to note that absence of cross-product terms between modes in Eq. (6) means that phase relationship between modes is not important in forming the mean-square pressure. For a linear time-invariant system, a stationary random input gives rise to a stationary random output. The above observation will establish a criterion for characterizing the sound field as being deterministic or random.

It is well known that a source at a corner can excite the maximum number of modes; this condition will be taken in the following analysis. (The extent to which different source positions can affect the results will be left to Sec. IV.—Computer Study.) For high frequencies, the number of normal modes being summed in Eq. (6) is still very large so that summation can be converted into integration over wavenumber space.⁷ Since only those modes whose characteristic frequencies lie within the bandwidth of interest are to be summed, the corresponding limits of integration over the wavenumber space must be for an interval Δk around k_c , the wavenumber of the center frequency of the band. The modal density dN/dk_N at wavenumbers around k_c is given by $k_c^2 V/2\pi^2$. Considering only the oblique modes, which dominate at high frequencies, ϵ 's may be set equal to 2 and Eq. (6) can be changed

$$\bar{p}_{\Delta\omega}^2(\mathbf{r}) = 8\pi C^3 \left(\frac{\rho Q}{V} \right)^2 \left(\frac{V}{\beta S} \right) \frac{k_c^2}{2\pi^2} \times \int_{\Delta k} \langle \cos^2(k_x x) \cos^2(k_y y) \cos^2(k_z z) \rangle dk_N \quad (7)$$

where $S = 2(L_x L_y + L_y L_z + L_z L_x)$ is the surface area of the chamber and $\langle \rangle$ denotes an average over the surface of an octant of a sphere of radius k_N and thickness dk_N in the wavenumber space. Changing from rectangular to spherical coordinate systems for the wavenumber space it follows:

$$k_x = k_N \sin\theta \cos\phi, \quad k_y = k_N \sin\theta \sin\phi, \quad k_z = k_N \cos\theta.$$

The $\langle \rangle$ average of Eq. (7) becomes

$$\frac{1}{4\pi} \int_0^{\pi/2} \int_0^{\pi/2} \{ 1 + \cos(2k_N x \sin\theta \cos\phi) + \cos(2k_N y \sin\theta \sin\phi) + \cos(2k_N z \cos\theta) + \cos(2k_N x \sin\theta \cos\phi) \cos(2k_N y \sin\theta \sin\phi) + \cos(2k_N x \sin\theta \cos\phi) \cos(2k_N z \cos\theta) + \cos(2k_N y \sin\theta \sin\phi) \cos(2k_N z \cos\theta) + \cos(2k_N x \sin\theta \cos\phi) \cos(2k_N y \sin\theta \sin\phi) \cos(2k_N z \cos\theta) \} \sin\theta d\theta d\phi \quad (8)$$

Equation (8) is similar to the integral obtained by Waterhouse⁹ when he formulated the problem in terms of the free-wave model. This is no surprise because what Waterhouse averages, in terms of the waves coming from different directions, is physically equivalent to what are being averaged over the wavenumber space arising from the summation of the different excited modes. Using Waterhouse's result for the integral, one obtains

$$8\langle \rangle = 1 + j_0(2k_N x) + j_0(2k_N y) + j_0(2k_N z) + j_0(2k_N \xi_1) + j_0(2k_N \xi_2) + j_0(2k_N \xi_3) + j_0(2k_N r), \quad (9)$$

where $j_0(a) = \sin(a)/a$,

$$\xi_1^2 = x^2 + y^2, \quad \xi_2^2 = x^2 + z^2, \quad \xi_3^2 = y^2 + z^2, \quad r^2 = x^2 + y^2 + z^2.$$

The integral of the type

$$\int_{k_1}^{k_2} \frac{\sin(k_N a)}{k_N a} dk_N,$$

required when Eq. (9) is substituted into Eq. (7) has been considered by Cook¹⁰ who concluded that it is equal to

$$\frac{\sin(ka)}{ka} \Delta k + \text{terms of the order of } \left(\frac{k_2 - k_1}{k} \right)^2 \Delta k,$$

where $k = (k_2 + k_1)/2$ and $\Delta k = k_2 - k_1$. It follows that for narrow-band analysis, the variation of the mean-square pressure normalized by the asymptotic value becomes

$$[\bar{p}^2(\mathbf{r}) / \bar{p}^2(\infty)]_{\Delta\omega} = 1 + j_0(2k_c x) + j_0(2k_c y) + j_0(2k_c z) + j_0(2k_c \xi_1) + j_0(2k_c \xi_2) + j_0(2k_c \xi_3) + j_0(2k_c r), \quad (10)$$

a result identical to that derived from the free-wave model for an incoherent reverberant field.⁹ With the proper choice of the variables x, y , and z , Eq. (10) can be considered as a representation of the interference pattern from a corner of a room.

Other interference patterns from edges and planes can also be derived from Eq. (10). For example, the interference pattern from the edge represented by the $0-x$ axis of the room is obtained by substituting the variables (x_1, y, z) in Eq. (10) where x_1 is the root point on the $0-x$ axis. Provided the root point is not too close to the corner, for high frequencies consideration, the following approximations:

$$j_0(2k_c x_1) = j_0(2k_c \xi_1) = j_0(2k_c \xi_2) = j_0(2k_c r) \approx 0,$$

will simplify Eq. (10) to

$$[\bar{p}^2(x_1, y, z) / \bar{p}^2(\infty)]_{\Delta\omega} = 1 + j_0(2k_c y) + j_0(2k_c z) + j_0(2k_c \xi_3), \quad (11)$$

which is again the same as that given by Waterhouse⁹ for the interference pattern from an edge. Similar argument will show that the interference pattern from the $x-z$ plane at the root point $(x_1, 0, z_1)$ which is not close to an edge or a corner is given by

$$[\bar{p}^2(x_1, y, z_1) / \bar{p}^2(\infty)]_{\Delta\omega} = 1 + j_0(2k_c y). \quad (12)$$

Equation (12) also agrees with Waterhouse's result.⁹

III. PURE-TONE EXCITATION

For a single source position the mean-square sound pressure under pure-tone excitation is given by Eq. (2). Argument based on the sign of the ψ 's alone is not sufficient to justify neglect of the cross-product terms between modes for this particular case. In other words, the phase relationship between modes has to be taken into account. Such a field is definitely not random. Further discussion about the coherent nature of this case will be given in a separate article.¹¹ It suffices to say that one cannot expect the free-wave model results to be applicable here. This conclusion is also verified by the computer study reported in Sec. IV.

The situation is quite different when averaging over source positions is applied. The cross-product terms between modes are effectively eliminated by the orthogonality properties of the characteristic function. With the help of

$$\int_V \int_V \psi_N(\mathbf{r}_0) \psi_M(\mathbf{r}_0) dx_0 dy_0 dz_0 = \begin{cases} 0, & \text{if } N \neq M, \\ V \Lambda_N, & \text{if } N = M, \end{cases} \quad (13)$$

Eq. (2) can be rewritten as:

$$[\bar{p}^2(\mathbf{r}, \omega)]_s = \frac{1}{2} \left(\frac{\rho \omega Q}{V} \right)^2 \sum_N \frac{\psi_N^2(\mathbf{r})}{\Lambda_N [(k^2 - k_N^2)^2 + (2k\delta_N)^2]}, \quad (14)$$

where $[\]_s$ signifies averaging over all possible source positions in the room. In other words an equivalent random field has been achieved by the source position averaging technique. Such a technique has also been used by Chien and Soroka³ in deriving the cross-correlation function.

Using the same high-frequency argument as before, it can be shown that Eq. (14) becomes, with $\delta = \beta S/V$,

$$[\bar{p}^2(\mathbf{r}, \omega)]_s = \frac{1}{2} \left(\frac{\rho \omega Q}{V} \right)^2 \frac{V}{2\pi^2} \int_0^\infty \frac{k_N^2 \langle \rangle}{(k^2 - k_N^2)^2 + (2k\delta)^2} dk_N, \quad (15)$$

where $\langle \rangle$ is given by Eq. (9). The modal density dN/dk_N at any wavenumber k_N is $k_N^2 V/2\pi^2$ and the limit of integration for this case is from 0 to ∞ because the summation in Eq. (14) is for all possible values of Chien and Soroka³ have shown that,

$$\int_0^\infty \frac{[1 + j_0(2k_N x)] k_N^2}{(k^2 - k_N^2)^2 + (2k\delta)^2} dk_N = [1 + j_0(2kx)] \pi / 4\delta. \quad (16)$$

Thus combining results from Eq. (15) and Eq. (16), the normalized mean-square pressure for pure-tone excitation with averaging over source positions can be written as:

$$[\bar{p}^2(\mathbf{r}, \omega) / \bar{p}^2(\infty, \omega)]_s = 1 + j_0(2kx) + j_0(2ky) + j_0(2kz) + j_0(2k\xi_1) + j_0(2k\xi_2) + j_0(2k\xi_3) + j_0(2kr), \quad (17)$$

which has the same form as Eq. (10). Thus it can be concluded that the free-wave and eigenmode models

are consistent for the case of pure-tone excitation with averaging over source positions.

IV. COMPUTER STUDY

It is recognized that analyses given in the previous two sections are high-frequency asymptotic solutions. A more direct approach by carrying out the actual summations of the allowable modes in Eqs. (2), (6), and (14) would be instructive and desirable. This type of calculation is currently being carried out using a Nova 820 minicomputer. An in-depth computer study of the reverberant sound field is beyond the scope of the present paper and will be presented later, but in the meantime some pertinent results will be given in this section to support the analytic analysis.

In this computer study, all types of modes were included except the dc component due to the (0,0,0) mode. For cases of pure-tone excitation with or without averaging over source positions, Eqs. (14) and (2) show that the main contribution to $\bar{p}^2$ comes from terms with $k_{N,M} \approx k$. The total number of modes to be summed was governed by an arbitrary chosen criterion that $|k^2 - k_{N,M}^2| \leq 100$, which turned out to be quite adequate. For the case of narrow-band analysis, the total number of modes being summed was dictated by the requirement that the modal frequency be within the bandwidth of interest. The bandwidth chosen was 1/3 octave. Since there is no way of establishing $\bar{p}^2$ in this study, the boundary points had to be tied to the theoretical values and values of all the other points to be normalized by the boundary points accordingly. Dimensions and absorption characteristics of the boundaries of the room were chosen to simulate conditions of the NRC big reverberation chamber and these are summarized in Table II. The Schroeder large room limit¹² for this room is slightly above 315 Hz.

Interference patterns in front of a plane boundary as well as from a corner have been computed for a large

TABLE II. Summary of room characteristics used in the computer study. Room dimensions: 8 m × 6.5 m × 4.9 m.

f	Wall absorption characteristics Specific normal admittance (β) ^a
125	0.0023
160	0.0030
200	0.0028
250	0.0027
315	0.0028
400	0.0030
500	0.0032
630	0.0033
800	0.0035
1000	0.0035
1250	0.0037
1600	0.0038
2000	0.0043

^aNote: For 1/3 octave narrow-band analysis, β has been assumed to be uniform. The smallest value of 0.0023 was used for all frequencies.

FIG. 1. Variance from free-wave model prediction of interference pattern in front of the x - z plane location (4, 0, 2.25) under different conditions: — 1/3 octave narrow-band, source at (0, 0, 0); - · - 1/3 octave narrow-band, source at (2, 1.5, 1); - - - pure-tone excitation with averaging over source positions; - · · - pure-tone excitation, source at (0, 0, 0).

number of frequencies extending from 100 Hz–2 kHz under three different conditions: pure-tone excitation with or without source position averaging and 1/3 octave narrow-band analysis with single source position. A total of 25 points were used for each interference pattern extending to $2kr = 12$. The computed results were compared with the theoretical prediction from the free-wave model and a variance was calculated to establish a degree of agreement.

Figure 1 shows results of the variance for interference patterns in front of the x - z plane at a location close to the midpoint of the plane for four different cases. As indicated by the solid curve, the best comparison between computer results and theory is achieved for the 1/3 octave narrow-band analysis with the sound source at a corner of the room. The agreement is very good down to 160 Hz as illustrated by the actual comparison between the computed pattern (open circles) and the theoretical curve shown in Fig. 2. Agreement deteriorates at low frequencies, however, when the sound source was not located at the corner as indicated by the dash-dot curve in Fig. 1. The reason being that most of the modes are no longer fully excited.

The dash-dot-dot curve of Fig. 1 shows no agreement between the computer results and prediction of the free-wave model at all frequencies for the case of pure-tone excitation with a fixed source position. This

FIG. 2. Computed interference patterns in front of the x - z plane at location $(4, 0, 2.25)$; $\circ$ 1/3 octave narrow band at 160 Hz, source at $(0, 0, 0)$; $\bullet$ pure-tone excitation at 2 kHz with averaging over source positions; $\bullet$ pure-tone excitation at 1.25 kHz, source at $(0, 0, 0)$.

confirms the statement made in Sec. III that the field is definitely not random for this case. A plot of the computed pattern (solid circles) at 1.25 kHz is shown in Fig. 2 to indicate the degree of disagreement. As pointed out in Sec. III also, an equivalent random field result can be approached by averaging over all possible source positions. The dash-dash curve of Fig. 1 suggests that this is true at least at high frequencies. This is more clearly demonstrated in Fig. 2 where the computed pattern (half solid circles) at 2 kHz is also plotted.

Similar conclusions can be drawn from the computer results of interference patterns from a corner. A few examples of the computed pattern are shown in Fig. 3 to show the degree of agreement with predictions from the free-wave model. Good comparison with theory is obtained even at 160 Hz for the case of 1/3 octave narrow-band analysis with the sound source at a corner, and reasonable agreement with theory can be achieved for the case of pure-tone excitation with averaging over many source positions at high frequencies.

V. CALCULATION OF TRUE ROOM-AVERAGED VALUE OF $\bar{p}^2$

The accepted procedure for obtaining a true room-average value of $\bar{p}^2$ in relation to sound power measurements in reverberation chambers^{13,14} is to sample the mean-square pressure in the central portion of the room at least $\lambda/2$ away from any boundaries and apply a correction factor to account for the buildup of $\bar{p}^2$ near the boundaries. Such a procedure has been recommended indiscriminately for cases with random-noise or pure-tone sources. In view of the new results presented here, it is worthwhile to reconsider this cor-

FIG. 3. Computed interference patterns from a corner along a line with $\theta = 0.8$ rad. and $\phi = 0.3$ rad. $\circ$ 1/3 octave narrow band at 160 Hz, source at $(0, 0, 0)$; $\bullet$ pure-tone excitation at 2 kHz with averaging over source positions; $\bullet$ pure-tone excitation at 1.25 kHz, source at $(0, 0, 0)$.

rection factor under the different conditions of excitation. A more direct approach will be taken in the following developments using the point source formulation.

A. Narrow-band noise analysis-single source position

The mean-square pressure at any location, $\mathbf{r}$, is given by Eq. (6). Applying Eq. (13), the true room-average value of $\bar{p}^2$, denoted by $\langle \bar{p}^2 \rangle_V$ can be written as:

$$\langle \bar{p}_{\Delta\omega}^2(\mathbf{r}_0) \rangle_V = \left(\frac{\rho Q}{V} \right)^2 \frac{\pi C^3}{8} \sum_N' \frac{\psi_N^2(\mathbf{r}_0)}{\Lambda_N \delta_N}. \quad (18)$$

On the other hand, if the sampling of $\bar{p}^2$ is to be done in the central region of the room $\lambda/2$ from all boundaries, the sample-mean value of $\bar{p}^2$ will be different from the true room-average value. Using the continuous sampling concept, the sample-mean value can be obtained by integrating Eq. (6) over the volume of interest, that is:

$$\langle \bar{p}_{\Delta\omega}^2(\mathbf{r}_0) \rangle_{V'} = \left(\frac{\rho Q}{V} \right)^2 \frac{\pi C^3}{8} \frac{1}{V'} \sum_N' \frac{\psi_N^2(\mathbf{r}_0)}{\Lambda_N \delta_N} \times \int_{V'} \int \int \psi_N^2(\mathbf{r}) dx dy dz, \quad (19)$$

where $V' = (L_x - \lambda)(L_y - \lambda)(L_z - \lambda)$. It can be shown that

$$\begin{aligned} \int_{V'} \int \int \psi_N^2(\mathbf{r}) dx dy dz &= \left\{ \frac{L_x}{2} - \frac{\lambda}{2} [1 + j_0(k_x \lambda)] \right\} \\ &\times \left\{ \frac{L_y}{2} - \frac{\lambda}{2} [1 + j_0(k_y \lambda)] \right\} \\ &\times \left\{ \frac{L_z}{2} - \frac{\lambda}{2} [1 + j_0(k_z \lambda)] \right\}, \end{aligned} \quad (20)$$

for k_x, k_y, k_z not equal to zero and $\{ \} = L_i - \lambda$ if $k_i = 0$. A correction factor, COR, can thus be computed using Eqs. (18), (19), and (20):

$$\text{COR}_{\Delta\omega} = \langle \bar{p}_{\Delta\omega}^2(\mathbf{r}_0) \rangle_V / \langle \bar{p}_{\Delta\omega}^2(\mathbf{r}_0) \rangle_{V'}. \quad (21)$$

FIG. 4. Comparison of computed boundary correction factors with Waterhouse's formula as a function of frequency for 1/3 octave narrow band.

The computation was carried out with a minicomputer as reported in Sec. IV. The results are shown in Fig. 4.

Figure 4(a) shows a good comparison between the current computer results and the approximate formula given by Waterhouse⁹ for the case where the source was located at a corner of the room. When the source was moved away from the corner, the correction factor at frequencies below the Schroeder large-room limit (around 315 Hz for this case) varied depending on the source position as indicated by results shown in Figs. 4(b) and 4(c). At high frequencies, the correction factor becomes independent of source position.

B. Pure-tone excitation with averaging over source positions

Similar derivation applied to Eq. (14) gives for the true-room average value,

$$\langle [\bar{p}^2(\omega)]_s \rangle_V = \frac{1}{2} \left(\frac{\rho \omega Q}{V} \right)^2 \sum_N \frac{1}{[(k^2 - k_N^2)^2 + (2k\delta_N)^2]}, \quad (22)$$

and for the sample-mean value,

$$\langle [\bar{p}^2(\omega)]_s \rangle_{V'} = \frac{1}{2} \left(\frac{\rho \omega Q}{V} \right)^2 \frac{1}{V'} \times \sum_N \frac{\{ \{ \} \} \{ \{ \} \}}{\Lambda_N [(k^2 - k_N^2)^2 + (2k\delta_N)^2]}, \quad (23)$$

where $\{ \{ \} \}$ is defined by Eq. (20). Thus a similar

FIG. 5. Comparison of computed boundary correction factors with Waterhouse's formula as a function of frequency for pure-tone excitation with averaging over source positions.

correction factor defined by

$$[\text{COR}(\omega)]_s = \langle [\bar{p}^2(\omega)]_s \rangle_V / \langle [\bar{p}^2(\omega)]_s \rangle_{V'} \quad (24)$$

can be computed for different frequencies.

Figure 5 depicts results obtained for this case. Very good comparison with Waterhouse's result⁹ is again achieved for frequencies above 315 Hz.

C. Pure-tone excitation-single source position

Applying Eq. (13) to Eq. (2), the true room-average value of $\bar{p}^2$ for this case becomes

$$\langle \bar{p}^2(\mathbf{r}_0, \omega) \rangle_V = \frac{1}{2} \left(\frac{\rho \omega Q}{V} \right)^2 \times \sum_N \frac{\psi_N^2(\mathbf{r}_0)}{\Lambda_N [(k^2 - k_N^2)^2 + (2k\delta_N)^2]}. \quad (25)$$

However, no easy solution has been found for the sample-mean value in region $\lambda/2$ away from the boundaries. Discrete sampling technique has to be applied to Eq. (2). A total of 343 randomly sampled values of $\bar{p}^2$ within the aforementioned region were used in calculating the sample-mean value of $\bar{p}^2$. Comparing this result with that obtained from Eq. (25) a correction factor was calculated. It turned out that because of the small number of finite sample values, there was too much scattering in the final results for a proper comparison with Waterhouse's formula. Increasing the number of sampled points would lengthen the computation time and there is no way of establishing in advance how many points will be sufficient. A compromise solution was taken. It is felt that a reasonable conclusion can be drawn if results of the present case are compared with results obtained for the case with averaging over source positions (Sec. VB) using the same sampling technique at the same sampled locations in the room. A comparison of results is shown in Table III for two different runs: one with the source at (0, 0, 0) and the other at (5.5, 2.5, 2).

It is interesting to note that although the interference patterns for this case of pure-tone excitation with single source position do not agree with theory (see Sec. IV.), the boundary correction factor concept for sam-

TABLE III. Comparison of boundary correction factors for the case of pure-tone excitation with and without averaging over source positions using 343 randomly sampled positions.

Frequency (Hz)	Run #1 ^a		Run #2	
	With [] _s ^b	Source at (0,0,0)	With [] _s	Source at (5.5, 2.5, 2)
125	1.77	2.14	1.34	1.44
160	1.54	1.55	1.52	1.19
200	1.33	1.25	1.20	1.05
250	1.21	1.28	1.23	1.10
315	1.08	0.91	1.01	0.90
400	1.31	1.28	1.08	0.99
500	1.24	1.16	1.14	1.21
630	1.07	1.03	1.16	1.22
800	1.16	1.08	1.06	1.11
1000	1.14	1.23	1.01	0.92
1250	1.06	1.01	1.11	0.99
1600	0.97	0.94	1.01	1.19
2000	1.00	0.93	0.98	0.89

^aNote: The randomly sampled positions between Runs #1 and #2 were different; this accounts for the differences in results for the case with averaging over source positions in the two runs.

^b[]_s = averaging over many source positions.

pling in a limited region still appears to be appropriate, possibly due to some sort of averaging effect as indicated by the reasonable agreement of results with the case of averaging over source positions.¹⁵ This result is significant in that it provides an acceptable argument for extending Waterhouse's boundary correction factor concept even to the case of a single source emitting a single frequency.

VI. CONCLUSION

At high frequencies, closed-form solutions for the interference patterns near boundaries have been obtained from an eigenmode analysis, and found to agree with those predicted from the free-wave model. The incoherent or random nature of the sound field required by the free-wave model can be achieved only with random-noise excitation. For pure-tone excitation, an equivalent quasirandom field can be approached by averaging over many source locations.

The analytical solutions are further supported by a computer study which reveals the departure of these interference patterns from predictions at low frequencies and/or for different source positions other than that at a corner of the room. The computer study confirms that the free-wave model prediction does not apply to the case of pure-tone excitation with single source position

A more direct approach based on the eigenmode model has been taken for investigating the boundary correction factor concept proposed by Waterhouse in obtaining

the true room-average value of $\bar{p}^2$. Two interesting results came out of this study: (1) The Schroeder large-room limit seems to set a low frequency limit above which the free-wave and eigenmode models agree in the case of 1/3-octave band analysis irrespective of source positions. Even for pure tone excitation a similar conclusion is reached, if averaging is done over many source positions (the computation were for all possible source positions). (2) The usual boundary correction factor also appears to be of the right order for the case of pure-tone excitation with single source position although the sound field is not random in this case. Current results seem to indicate that the correction factors are similar to those for the case with averaging over source positions. Thus to improve the low-frequency situation, an obvious solution, both for cases of pure-tone and random noise excitation is to average over several source positions.

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¹⁵The author would like to thank David Lubman for his suggestion to look into this possibility.