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by D. E. Allen, J. H. Rainer and G. Pernica

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Vibration criteria for assembly occupancies

D. E. ALLEN, J. H. RAINER, AND G. PERNICA

Division of Building Research, National Research Council of Canada, Ottawa, Ont., Canada K1A 0R6

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A design procedure based on user acceptability of vibration is presented for floor structures that are subjected to rhythmic activities such as dancing and jumping exercises. The procedure is applied to different occupancies and floor constructions using available information on dynamic loading; the results are summarized as a table of minimum natural frequencies. The procedure is contained in the Supplement to the National Building Code of Canada 1985.

L'article présente une méthode de calcul qui, à partir d'un niveau de vibrations choisi, vérifie les planchers devant servir à des activités rythmiques telles que la danse et les sauts. La méthode est appliquée à différents cas d'utilisation et de construction de planchers en se servant des informations disponibles sur le chargement dynamique. Les résultats sont réunis dans un tableau de fréquences naturelles minimales. La méthode est incluse dans le supplément du Code national du bâtiment du Canada de 1985.

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Introduction

As floor spans of buildings have gradually increased, the natural frequencies of floor structures have decreased to the point where they approach or equal the forcing frequencies of rhythmic activities such as dancing, exercising, and audience participation. The large vibrations that have sometimes resulted have annoyed occupants and owners alike and occasionally have been of concern for reasons of safety (Bachmann 1984; Matthews *et al.* 1982; New Civil Engineer International 1981; Pernica 1983; Rainer 1984; Rainer and Allen 1973; Vannoy and Heins 1979). To counteract this, the Commentary on the National Building Code of Canada (NBC) has, since 1970, recommended a minimum natural frequency of 5 Hz for floor structures subjected to rhythmic activities; in 1975 it recommended 10 Hz for very repetitive activities such as dancing. The incidence of vibration problems has, nevertheless, increased. The 1985 NBC will introduce a new clause requiring a dynamic analysis when the fundamental natural frequency of a floor structure that may be subjected to rhythmic activities is less than 6 Hz. Guidance will be given in Commentary A, Chapter 4, of the Supplement to the National Building Code of Canada (1985).

The NBC requirement for a dynamic analysis when the natural frequency is less than 6 Hz is primarily related to safety, i.e., to overloading or fatigue. Floor vibration, however, is much more likely to annoy people than to cause either overloading or fatigue. There-

fore the 1985 NBC Commentary contains a design procedure to ensure that vibration of floor structures is acceptable to most users. This paper provides additional information on the design procedure.

Derivation of design criterion

Activities that can cause significant dynamic loads include dancing, jumping, marching, running, and foot stamping. The force variations with time for these activities have not yet been well defined, but in the main they consist of regular pulses resembling full or truncated sinusoids at the rhythmic frequency, f , equal to the beat of music or some other timing device. Deviations from pure sine waves appear as higher harmonics or multiples of the rhythmic frequency, f . If w_p = weight of people on the floor, the forcing function, $P(t)$, will be given by the sum of the static component, w_p , and a dynamic component, $F(t)$, represented as follows:

$$[1] \quad P(t) = w_p + F(t) \\ = w_p \left[1 + \sum_{n=1}^{\infty} \alpha_n \sin(2\pi nft + \phi_n) \right]$$

where α_n is the dynamic load factor corresponding to the n th harmonic for the activity, and ϕ_n is the phase angle. It may be seen that α_n is the Fourier coefficient of the normalized dynamic loading function. In the derivation that follows only the fundamental component, $n = 1$, is used, since this is generally the dominant component of the dynamic loading function. Higher harmonics can, however, be treated similarly, as will be discussed. Thus

NOTE: Written discussion of this paper is welcomed and will be received by the Editor until December 31, 1985 (address inside front cover).

FIG. 1. Peak acceleration versus frequency ratio ([9]).

$$[2] \quad F(t) = \alpha w_p \sin 2\pi ft$$

For determining dynamic response to the sinusoidal loading of [2] the floor structure can be modelled by means of an equivalent single-degree-of-freedom beam oscillator. The maximum dynamic deflection, Δ_d , of such an oscillator under steady-state response is given by

$$[3] \quad \Delta_d = \rho \Delta_s = \rho \alpha w_p / k$$

where k is the stiffness of the beam oscillator, Δ_s is the static deflection due to the dynamic component of load, αw_p , and

$$[4] \quad \rho = \frac{1}{\sqrt{\left[1 - \left(\frac{f}{f_0}\right)^2\right]^2 + \left(2\beta \frac{f}{f_0}\right)^2}}$$

is the dynamic amplification or response factor (Timoshenko *et al.* 1974). In [4], f is the forcing frequency, f_0 the natural frequency, and β the damping ratio of the beam oscillator. Under resonance conditions ($f = f_0$) the response becomes very large after building up over a number of cycles.

The peak acceleration, a_m , is determined from [3] as follows:

$$[5] \quad a_m = (2\pi f)^2 \Delta_d = (2\pi f)^2 \rho \Delta_s$$

Thus, the peak acceleration can be determined by means of [3]–[5] from the dynamic component of loading, forcing frequency, stiffness of the beam oscil-

lator, and its natural frequency. Stiffness and natural frequency are, however, interrelated. The fundamental natural frequency of a simply supported beam of span L , stiffness EI , and weight per unit length w_l is given by (Timoshenko *et al.* 1974)

$$[6] \quad f_0 = \frac{9.87}{2\pi} \sqrt{\frac{EIg}{w_l L^4}}$$

where g is acceleration due to gravity. The static mid-span deflection, Δ_s , under uniform load, αw_p , is

$$[7] \quad \Delta_s = \frac{5}{384} \frac{\alpha w_p L^4}{EI}$$

Thus

$$[8] \quad f_0 = \frac{1}{2\pi} \sqrt{\frac{1.3g \alpha w_p}{\Delta_s w_l}}$$

Substitution in [5] of expressions for Δ_s and ρ determined from [8] and [4] gives

$$[9] \quad \frac{a_m}{g} = \frac{1.3 \alpha w_p / w_l}{\sqrt{\left[\left(\frac{f_0}{f}\right)^2 - 1\right]^2 + \left[2\beta \frac{f_0}{f}\right]^2}}$$

Equation [9] can be used to estimate peak acceleration and hence to evaluate a floor structure subjected to sinusoidal loading. Figure 1 shows how peak acceleration from [9] varies with frequency ratio, and that outside a band surrounding the resonance condition, $f_0 = f$, [9] can be approximated by assuming $\beta = 0$. Inversion of [9] then leads to a minimum frequency ratio above resonance that provides the following criterion for design:

$$[10] \quad f_0 \geq f \sqrt{1 + \frac{1.3 \alpha w_p}{a_0/g w_l}}$$

where a_0 is a limiting acceleration.

The derivation of [9] and [10] has been carried out for simply supported beam-like floor construction. A similar derivation applies to other configurations such as cantilevers, simply supported plates, and fixed-end beams or plates. If [6] and [7] are replaced by appropriate formulae for these configurations, then the constant 1.3 in [8]–[10] for simply supported beams is 1.3 for fixed-end beams, 1.5 for fixed cantilevers, and 1.6 for a square plate simply supported or fixed; all other parameters remain the same. Equations [9] and [10] therefore apply reasonably well for most floor systems.

By means of [8], [10] can be expressed in terms of a limitation on static deflection, Δ_s , rather than natural frequency. This is done in the Supplement to the National Building Code (1985), but [10] is preferred for the following reasons: (1) As will be discussed, the major safety-and-serviceability-related problem is one of resonance between the source of excitation and the

TABLE 1. Suggested vibration limits for activities on floors: 1.5–8 Hz

Activity or occupancy	Limiting peak acceleration a_0/g	Reiher and Meister rating at 3 Hz	ISO duration for reduced comfort, h
Threshold of perception	0.001–0.002	Just perceptible	24
Offices and residences (daytime)	0.005	Perceptible	24
Dancing and dining	0.02	Annoying	4
Physical exercise, sports, and lively concerts (gymnasias, arenas, stadia)	0.05	Unpleasant	1

(a) TIME RECORD LOW PASS-FILTERED AT 9 Hz

(b) FOURIER AMPLITUDE SPECTRUM

range is an acceleration of the order of 0.001–0.002 g (Reiher and Meister 1931; International Organization for Standardization 1978). Intermittent vibrations of up to 0.005 g have been found tolerable in offices and residences during the daytime (Allen and Rainer 1976). Accelerations of 0.02 g have been found acceptable or tolerable for people sitting or standing around a dance floor (Matthews *et al.* 1982), but would be unacceptable in a home or office environment. For lively concerts or sporting events 0.05 g is suggested as the tolerance limit for spectators who are not directly participating in the activity. Suggested limiting values are summarized in Table 1, together with a descriptive label corresponding to the Reiher and Meister (1931) scale and the ISO rating in hours for “reduced comfort” (International Organization for Standardization 1978). Because of the subjective nature of human reaction (a few people like floor vibration; others get used to it), a limiting acceleration is difficult to establish accurately.

Estimation of dynamic load parameters

Experience with field measurements indicates that the significant range of rhythmic frequency, f , is 1.5–3.5 Hz. Beyond 3 Hz it becomes difficult for a large group to maintain a coordinated rhythm.

The dynamic load factor, α , can be determined directly from force records, but so far only a few values are available. Some force records and their Fourier spectra, obtained from dynamic tests on a large platform at the Division of Building Research, National Research Council of Canada, are shown in Figs. 2–4. For eight people exercising by jumping (Fig. 2) the time record shows a nearly flat portion at the bottom of the record corresponding to zero load, when the participants lose contact with the floor. The corresponding Fourier spectrum in Fig. 2b shows that higher harmonics of the rhythmic frequency of the jumping exercise are present. The dynamic load factor was determined to be 1.4 for the first harmonic at 2.1 Hz and

FIG. 2. Floor forces produced by eight people jumping: (a) time record low pass-filtered at 9 Hz, (b) Fourier amplitude spectrum.

floor, so that it is important for the designer to determine the natural frequency of the floor. (2) A static calculation of Δ_s under uniform load, αw_p , for beams continuous over supports can be seriously in error; a continuous beam of equal spans on pin supports, for example, should be treated as simply supported because the vibration nodes are at the supports.

The application of [10] for design, as well as the estimation of its parameters, will be discussed.

Estimation of limiting acceleration

User reaction and tolerance to vibrations vary with the individual, activity, duration, and surroundings. The threshold of perception in the 1.5–8 Hz frequency

FIG. 3. Floor forces produced by one person jumping: (a) time record low pass-filtered at 9 Hz, (b) Fourier amplitude spectrum.

0.6 for the second harmonic at 4.2 Hz. Figure 3 shows similar results for one person jumping for exercise at 2.1 Hz, where the dynamic load factor was determined to be 1.5 for the first harmonic and 0.5 for the second; additional harmonic components occur at multiples of half the rhythmic frequency owing to a slightly larger force at every second cycle of the exercise.

Figure 4 shows results for eight people on the test platform, slowly running in step with a footfall frequency of 2.3 Hz. It indicates a more regular sinusoidal response and a virtual absence of higher harmonics in the spectrum of Fig. 3b. For this activity, α was determined to be 1.0 for the first harmonic and 0.1 for the second.

The dynamic load factor, α , can also be estimated from field measurements on actual floors by means of [9] if the physical parameters and distribution of dynamic loads are known; a few examples of calculated values of α from field measurements are presented in Table 2. Equation [9] relates to response at the forcing frequency only, so that other measured frequency components had to be removed by filtering. In a number of cases the magnitude and distribution of the load, w_p , had to be estimated or extrapolated, so that calculated values of α are somewhat approximate. Nevertheless, as may be seen in Table 2, they are in general agreement with the few values of α that were measured directly. Resonance cases were not included because of the large uncertainty in estimating dynamic

FIG. 4. Floor forces produced by eight people running slowly: (a) time record low pass-filtered at 9 Hz, (b) Fourier amplitude spectrum.

amplification.

On the basis of these results, suggested values of f , w_p , and α suitable for design purposes are given in Table 3. To account for higher harmonics from jumping exercises, Table 3 includes a dynamic load factor of 0.25 for forcing frequencies up to 6 Hz. The vibrations from the first harmonic (e.g., $f = 3$ Hz) and the second harmonic (e.g., $f = 6$ Hz) should be combined, but because of the scarcity of loading information and the adoption of conservative assumptions for Table 3, it is recommended that each case be treated separately and the governing one chosen. It is expected that more information will become available from research now in progress.

Design procedure for assembly occupancies

The following procedure is proposed for checking the design of floors for assembly occupancies subject to coordinated rhythmic activities. It may be necessary to check more than one type of activity and loading case.

(1) Determine the type of activity, estimate the density of occupancy, and obtain the distributed weight of participants, w_p ; see Table 3 for guidance. When only a portion of the span is occupied by the activity (e.g., a designated dance area), the load w_p may be estimated by taking the total load on the partially loaded span and distributing it uniformly over the full span. A more accurate method of determining w_p is to calculate the uniform loading that produces the same centre span

TABLE 2. Dynamic load factor, α , from field tests and direct load measurements

Activity	f , Hz	f_0 , Hz	w_p , kPa	w_t , kPa	a_m , g	α	Reference
Field tests							
Dance—rock & roll	2.3	3.0	0.35	2.7	0.10	0.42	Vannoy and Heins 1979
Dance—rock & roll	3.5	5.0	0.3	2.7	0.05	0.36	Rainer and Allen 1973
Dance—rock & roll	3.3	5.0	0.60	3.1	0.02	0.10	Matthews <i>et al.</i> 1982
—polka	2.2	5.0	0.47	3.0	0.007	0.14	
Dance—polka	2.3	5.6	0.25	2.8	0.009	0.38	*
Jumping exercises	2.45	4.9	0.4	4.0	0.05	1.2	Bachmann 1984
Selected gymnastic	2.14	3.45	0.075	7.2	0.013	1.6	Rainer 1984
exercises	3.03	3.45	0.075	7.2	0.030	0.71	
Direct load measurements							
Jogging	2.5					1.0	
Jumping exercises	2.1					1.5	
Foot stamping	2–3					0.2	

*G. A. Trusler, private communication.

TABLE 3. Suggested design parameters for rhythmic events

Activity	Forcing frequency f , Hz	Weight of participants* w_p , kPa	Dynamic load factor† α	Dynamic load αw_p , kPa
Dancing	1.5–3	0.6 (2.5 m ² /couple)	0.5	0.3
Lively concert or sports event	1.5–3	1.5 (0.5 m ² /person)	0.25	0.4
Jumping exercises	1.5–3	0.4 (2 m ² /person)	1.5	0.6
(second harmonic)	3–6	0.4 (2 m ² /person)	0.25	0.1

*Density of participants is for commonly encountered conditions. For special events the density of participants can be greater.

†Values of α are based on commonly encountered events involving a minimum of about 20 participants. Values of α should be increased for well-coordinated events (e.g., jump dances) or for fewer than 20 participants.

deflection as the actual partial loading.

(2) Choose an appropriate forcing frequency, f , and dynamic load factor, α ; see Table 3 or other appropriate sources. For jumping exercises, both the first harmonic (1½–3 Hz) and second harmonic (3–6 Hz) should be considered.

(3) Choose an acceptable limiting acceleration, a_0 , at the centre of the floor; see Table 1 or choose other appropriate limits. When the perceived location of vibration is not at the centre of the span, this can be taken into account by multiplying the acceleration criterion, a_0 , by $1/\sin(\pi x/L)$ where x is the location of perceived vibration and L is the span length.

(4) Estimate the total floor load, w_t (i.e., dead load + w_p), and determine the lowest acceptable fundamental frequency of the floor from [10]. For jumping exercises, pick the higher frequency for the two cases considered.

(5) Determine the fundamental natural frequency, f_0 , of the floor structure. In calculating f_0 the designer

should include the weight of participants, furniture, etc., in addition to that of the floor structure itself.

(6) If the criterion eq. [10] is not satisfied, indicating unsatisfactory performance, options include stiffening the structure, relocating the activity, controlling the activity (i.e., f and αw_p), and, in consultation with the owner, accepting a higher limiting acceleration.

The design criterion, [10], has been applied to typical cases in Table 4 and the results are summarized in Table 5. The minimum fundamental frequencies, f_0 , calculated for human reaction range from 5 Hz, for heavy floors, to 12 Hz, for light floors. These frequencies are often greater than 6 Hz, below which the NBC requires a dynamic analysis. This occurs because the NBC requirement is related primarily to safety and no serious safety-related problems are known to have occurred for floors with fundamental frequencies higher than 6 Hz. Cases that do not satisfy the minimum frequencies of Table 5 should be evaluated more carefully.

TABLE 4. Application of vibration criterion to different activities and floor constructions

Type of floor construction	Dead load, kPa	Forcing frequency f , Hz	Weight of participants w_p , kPa	Dynamic load αw_p , kPa	Total weight w_t , kPa	Minimum structural frequency f_0 ([10]), Hz
Dancing and dining, $a_0 = 0.02 g$						
Solid concrete	5	3	0.6	0.3	5.6	6.4
Steel joist	2.5	3	0.6	0.3	3.1	8.1
Wood	0.7	3	0.6	0.3	1.3	12.0
Lively concert or sports event, $a_0 = 0.05 g$						
Solid concrete	5	3	1.5	0.4	6.5	4.8
Steel joist	2.5	3	1.5	0.4	4.0	5.7
Wood	0.7	3	1.5	0.4	2.2	7.2
Jumping exercises, $a_0 = 0.05 g^*$						
Solid concrete	5	6	0.4	0.1	5.4	7.3
Steel joist	2.5	6	0.4	0.1	2.9	8.3
Wood	0.7	3	0.4	0.6	1.1	11.7

*Criterion is applied to two cases ($f = 3$ Hz and $f = 6$ Hz in Table 3) and the governing case selected.

TABLE 5. Minimum recommended natural floor frequencies for serviceability (based on Table 4)

Type of floor construction	Minimum natural frequency for type of occupancy, Hz	
	Dance floors,* gymnasias†	Stadia, arenas†
Solid concrete	7	5
Steel joist — concrete slab	9	6
Wood	12	8

*Limiting peak acceleration 0.02 g.

†Limiting peak acceleration 0.05 g.

Resonance considerations

When the rhythmic frequency, f , of the dynamic loading function coincides with or is near a natural frequency of the floor, a condition of resonance exists, and large vibration amplitudes will result. Such a coincidence of frequencies is particularly severe for the fundamental mode of vibration since all points on the floor have the same phase as the loading function. For a fully loaded floor, resonance at higher modes of vibration will not be so severe because out-of-phase components of higher mode shapes are counteracted by the loading function. If the floor is only partially loaded, however, and this loading pattern coincides substantially with portions of the mode shape having the same phase, then large amplifications can also be expected. This possibility should be considered for higher modes of a floor structure with a very low fundamental frequency that is less than the rhythmic frequency.

FIG. 5. Fourier amplitude spectrum of floor acceleration, a , during jumping exercise (Rainer 1984).

A condition of resonance can also exist if a higher harmonic of the rhythmic frequency coincides with a natural frequency of the floor. As was previously stated, the situation is generally most severe for the fundamental mode of vibration. Even a small dynamic component with forcing frequency equal to a natural frequency can cause significant response, since dynamic amplifications of 10 or more can occur at resonance (see Fig. 1). For a gymnasium floor subjected to a jumping exercise (Bachmann 1984) a large resonance response with accelerations up to 0.5 g occurred at 4.9 Hz, which is equal to twice the rhythmic frequency. This compares with accelerations of 0.05 g for the component of floor vibration at the rhythmic frequency of 2.45 Hz (see reference to "Bachmann 1984," Table 2). Figure 5 shows the spectrum of floor motion obtained from field measurements during rhythmic exercises (Rainer 1984). The response at the rhythmic frequency 2.3 Hz was smaller than that at 3.5 Hz (the fundamental frequency) and that at 4.6 Hz (a natural frequency of a higher mode), indicating the importance of resonance.

The harmonic loading at 3.5 Hz probably results from uneven forces similar to those that occur in Fig. 3.

Summary and conclusions

A design procedure based on user acceptability of vibration is presented for floor structures subjected to rhythmic activities. The method can be applied for dynamic loads produced by dancing, physical exercises, and audience participation at concerts and sporting events. Values for dynamic load parameters and acceleration limits are suggested for various activities, based on available information. Using these suggested values, application of the design procedure results in a set of minimum natural frequencies for different occupancies and floor constructions (Table 5). The minimum frequencies are generally higher than 6 Hz, below which a dynamic analysis is required by the 1985 NBC, for reasons of safety. Cases that do not satisfy the minimum frequencies of Table 5 should be evaluated more carefully.

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List of symbols

- a_m maximum acceleration under a sinusoidal loading function
- a_0 limiting acceleration
- f rhythmic frequency of activity, Hz
- f_0 fundamental natural frequency of floor structure, Hz
- g acceleration due to gravity
- k beam stiffness
- nf n th harmonic of rhythmic frequency
- w_p weight per unit area of participants in a rhythmic activity, kPa
- w_t total weight per unit area of structure, participants, etc., kPa
- α dynamic load factor assuming a sinusoidal loading function, see [2]
- α_n dynamic load factor for n th harmonic of the loading function, see [1]
- Δ_s static deflection due to a load αw_p
- ρ dynamic response factor, see [4]
- β damping ratio, fraction of critical