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ESTIMATING TIME TO FAILURE OF AGEING CAST IRON WATER MAINS UNDER UNCERTAINTIES

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Abstract
Water distribution networks form essential components of water supply systems in most urban centres. Water mains buried in the soil/backfill are exposed to different deleterious reactions and as a result, their design factors of safety may significantly degrade with time, leading to structural failure.

In most cases, a combination of circumstances leads to the failure of a pipe. Factors contributing to pipe failure include operational conditions, design parameters, external loads (traffic, frost, etc.), internal loads (operating and surge pressures), temperature changes, loss of bedding support, pipe properties and condition, and corrosion pit geometry. These are recorded rarely, if at all and it is therefore very difficult to ascertain the precise causes of failure. Even if all this information were available, any attempt to estimate the pipe condition state would involve considerable uncertainty due to large spatial and temporal variability that is inherent in this information. Estimation of time to failure is further exacerbated by the uncertainties in determining future corrosion rates.

In this paper, corrosion models and a previously developed analytical model based on Winkler-type pipe-soil interaction are used to estimate time to failure. Since available data are insufficient to establish credible probability distributions, uncertainties in the input data/parameters are handled using possibility theory and fuzzy arithmetic. Sensitivity analyses are carried out to identify the critical data/parameters that merit further investigation.

Keywords
Water mains, corrosion models, pipe-soil interaction model, factor of safety, time to failure, sensitivity analysis.

1 INTRODUCTION
The deterioration of aging water mains (pipes) is of increasing concern to all stakeholders, namely, water utilities (owners) as well as their customers. Their ability to deliver safe potable water without major interruptions is paramount. It is estimated that the financial costs to water utilities (public or private owners) for their repair and replacement exceeds $1 billion annually in Canada alone. The implementation of a proactive asset management strategy is essential to maintain water distribution networks that are both reliable and safe, given the limited availability of financial resources. In the past few years, different non-destructive techniques (NDT) have become available ([1], [2]) to measure remaining wall thickness, corrosion pits (ductile iron) or graphitization depths (cast iron) in pipes. Results obtained from these NDT measurements have to be incorporated within a broad decision support tool to assess condition state, determine time to failure and remaining service life for each inspected pipe and subsequently establish proactive management strategies.

In most cases, a combination of circumstances leads to the failure of a pipe. Factors contributing to pipe failure include operational conditions, design parameters, external loads (traffic, frost, etc.), internal loads (operating and surge pressures), temperature changes, loss of bedding support, pipe properties and condition, and corrosion pit geometry. This information is recorded rarely, if at all and it is therefore very difficult to ascertain the precise causes of failure. Even if all this information were available, any attempt to estimate the pipe condition state would involve considerable uncertainty due to large spatial and temporal variability that is inherent in this information. Estimation of time to failure is further exacerbated by the uncertainties in determining future corrosion rates. Rajani et al. [3] and Rajani and Tesfamariam [4] have developed a Winkler-type pipe-soil interaction (WPSI) model that accounts for most of the factors identified earlier as the predominant contributors to pipe failure. Tesfamariam et al. [5] have recently transformed these models into a possibilistic framework to incorporate the uncertainties discussed earlier. This paper describes further development of this fuzzy based model to integrate corrosion rates with the remaining wall thickness or pit geometry measurements obtained from NDT inspections to arrive at the time to failure. The development described here is a step to estimating remaining service life of one pipe length with several corrosion pits of significant depths observed at the time of inspection.

Existing physical deterministic models for pipe-soil interaction provide point estimates for the factor of safety (FS) and time to failure of pipes. These models also do not allow the identification of predominant contributory
factors that explain a specific failure mode that accounts for the uncertainties identified earlier. Therefore, the models need to be further developed to include uncertainties in order that the ‘probability’ or ‘possibility’ of pipe failure can be quantified. Possible approaches are Monte Carlo simulations, first order reliability methods, probability bound analysis (random sets) and possibilistic analysis using fuzzy arithmetic. In this paper, a possibilistic approach using fuzzy arithmetic is pursued to include uncertainties in the operational factors, the input data and the model parameters. Sensitivity analysis using rank-correlation coefficients is carried out to identify and evaluate the contributions of critical factors/parameters to the factor of safety. This paper reveals that the growth rate of a single corrosion pit in small diameter cast iron mains is nearly always more detrimental to thin (small diameter) than to thick (large diameter) pipes if all other data or properties remain unchanged.

2 PIPE-SOIL INTERACTION MODELS

Rajani and Tesfamariam [4] recently developed a Winkler-type pipe-soil interaction model that accounts for the unsupported length (likely to develop as a result of prolonged leakage or wash out) and soil elasto-plasticity. Axial, flexural and circumferential stress responses obtained from these models were consolidated with the previously reported ([3], [6]) responses, to establish an overall behaviour of buried water mains under the influence of earth and live loads, water pressure, temperature differential, unsupported length and pipe-soil interaction. Tesfamariam et al. [5] fuzzified these WPSI models to account for (a) uncertainties present in the input data and model parameters, and (b) failure modes using well accepted failure criteria for cast iron to determine the fuzzy factor of safety. Only sufficient details on the developed model and failure criteria for cast iron are given here to illustrate the extension of the model to include corrosion analysis of cast iron mains.

In general, the stress components due to external loads ($\sigma_x^w$), internal pressure ($\sigma_y^p$), temperature differential ($\sigma_x^T$) and longitudinal bending ($\sigma_x^f$) that contribute to the total axial stress ($\sigma_x^T$) are,

$$\sigma_x^T = \sigma_x^w + \sigma_y^p + \sigma_x^T + \sigma_x^f$$

Similarly, the total hoop stress ($\sigma_y^{total}$) has contributions from external loads ($\sigma_y^w$), internal pressure ($\sigma_y^p$), temperature differential ($\sigma_y^T$) and longitudinal bending ($\sigma_y^f$),

$$\sigma_y^{total} = \sigma_y^w + \sigma_y^p + \sigma_y^T = \sigma_y^w + \sigma_y^p + \sigma_y^T - \nu_p \sigma_x^f$$

where $\nu_p$ is the Poisson’s ratio for the pipe material. The thermal axial ($\sigma_x^T$) and hoop ($\sigma_y^T$) stresses in (1) and (2) are a consequence of temperature differential ($\Delta T$) between the inside of the pipe and the surrounding soil. Each of the stress components in (1) and (2) can be estimated using solutions provided in the above-cited references. It should be noted that the calculations of these stresses are applicable independent of pipe size.

3 FAILURE THEORIES

In-service water mains or pipes are subjected to continuous deleterious reactions, and internal and external loads that degrade the design factor of safety (FS). Consequently, the time to failure is significantly reduced if existing stresses on structurally deteriorated pipe exceed the expected or admissible design loads or stresses. The structural $FS$ of a deteriorating pipe diminishes ([4], [7]) as corrosion or graphitization pits initiate and subsequently grow over time, even though external and internal loads may not have changed significantly between time of installation and just prior to time of failure. Pipe failure is defined as an event in which the FS falls below a critical value, $FS_{cr}$, (usually set to 1), i.e., $FS < FS_{cr}$. Cast iron, a brittle material, typically fails through fracture at strains of 0.5%, rather than through yielding. Therefore, its failure is dictated by its ultimate strength. In contrast, yield strength is used to describe failure of ductile materials like those in ductile iron mains.

Two specific failure criteria (theories) are applicable to cast iron, namely, in-plane ($ip$) and bi-axial distortion energy (de) [5]. Only the distortion energy criterion is discussed here since the predominant stresses on small diameter mains are biaxial as opposed to in-plane stresses in large diameter mains.

The existence of bi-axial stresses requires the use of cast iron fracture criteria that consider both hoop and axial stresses. The failure criterion proposed by Coffin [8] and Fisher [9] and later modified by Mair [10] was selected for its robustness and simplicity. These researchers investigated the response of cast iron in the presence of graphite flakes. Typically, the ferrite-graphite matrix transmits the load when the cast iron is in compression; however, the graphite flakes act as stress raisers when subjected to tension. Consequently the ultimate tensile strength of a cast iron pipe is substantially lower than its ultimate compressive strength. Mair [10], based on his experimental work and that of Coffin [8] and Fisher [9], arrived at the conclusion that the cast iron failure criterion is best represented by the distortion energy theory developed by von Mises. This theory states that, failure by yielding or fracture occurs when the distortion energy per unit volume in a state of combined stress at any point in the body becomes equal to (or exceeds) that associated with yielding or fracture in a simple tension
The distortion energy failure criterion for bi-axial stresses written in terms of the factor of safety, $FS_{de}$, is as follows:

$$K\sigma = \frac{\sigma_a}{FS_{de}} \quad \text{for } \sigma_i > 0$$

$$\left(K\sigma_i\right)^2 - K\sigma_i\sigma_2 + \sigma_2^2 = \left(\frac{\sigma_a}{FS_{de}}\right)^2 \quad \text{for } \sigma_i > 0 \text{ and } \sigma_2 < 0$$

where $K$ is a stress concentration factor ($K = 3$ for $\sigma_i(i=1,2)$ in tension; $K = 1$ for $\sigma_i(i=1,2)$ in compression), $\sigma_i$ are the principal ($\sigma_i^a$ and $\sigma_i^{Total}$) stresses. The stress concentration factor, $K$, represents an aggregate reduction in tensile strength in the presence of carbon flakes, which act as stress raisers. It should therefore not be confused with a factor typically used to calculate stresses in the presence of defects with specific geometry, although it has the same connotation.

The ultimate strength of cast iron referred to in (3) is pertinent to cast iron that has no defects or corrosion (or graphitization) pits. The size of corrosion pits diminishes the intrinsic material strength of cast iron to the so-called residual ultimate strength ($\sigma_{w}$) in accordance with fracture mechanics theory ([11]) as follows,

$$\sigma_{w} = \alpha K_q / \beta \left[d / t \sqrt{a_n}\right]^n$$

where $\alpha$ and $s$ are constants used in the fracture toughness equations; $K_q$ is a provisional fracture toughness (MPa\(\sqrt{m}\)); $d$ is corrosion pit depth (mm); $t$ is pipe wall thickness (mm); $a_n$ is lateral dimension (mm) of a pit = $Ld$ (multiplier $L$ can be judged to have a value in the range of 1-5 in the absence of data); $\beta$ is the geometric factor for a double-edged notched tensile specimen, $\beta = a_1 (d / t)^{0.5}$, and $a_1$ and $b_1$ are constants for determining the geometric factor. Rajani et al. [11] established fracture toughness and other related parameters ($K_{q}, d, a_1, b_1$) based on limited experimental tests conducted on cast iron specimens taken from pipe samples.

4 CORROSION ESTIMATES

The loss of pipe wall thickness due to corrosion can be relatively uniform or localized. The rate of wall thickness loss has been the subject of debate, where it has been assumed to be constant or otherwise ([12], [13]). The rate of corrosion in uncoated cast iron pipes is generally higher at early age than in later years. There is evidence to suggest that corrosion is a self-inhibiting process, whereby as it proceeds, the protective properties of its products (generally iron oxides) reduce the corrosion rate over time ([12]). Models ([11], [14], [15], [16]) to predict pit depth growth range from linear to exponential functions. The two-phase (in the first phase a rapid exponential pit growth and in the second a slow linear growth) model proposed by Rajani et al. [11] is used here to estimate time to failure since it accommodates the self-inhibiting process discussed earlier.

$$d = a + kc \tau$$

$$\dot{d} = a + kce^{-ct}$$

where $\tau$ represents time. The constant combination of $a + kc$ represents the maximum corrosion rate and the constant $a$ corresponds to the minimum (usually terminal) corrosion rate. The constant $c$ can be viewed as an ‘inhibition’ factor. Metal and soil corrosivity (properties) will dictate the values of constants $a$, $c$, and $k$. Values for constants $a$, $c$, and $k$ were selected based on observations that through corrosion pits in small diameter cast iron mains can occur as early as 40 years (in very high corrosivity soils) and as late as 100 to 120 years or never (in very low corrosivity soils) as depicted in Fig. 1 and Table 1. Wall thickness range for 150 to 200 mm (6" to 8") cast iron mains operating at pressures between 345 and 690 kPa (50 and 100 psi) is typically between 11.2 and 15.2 mm (0.44" and 0.62").

5 UNCERTAINTY ANALYSIS AND ITS INTEGRATION WITH PIPE-SOIL INTERACTION MODELS, MATERIAL FAILURE THEORIES AND CORROSION

Complex models often involve input data or parameters with uncertainties, which are best represented by random variables with known or assumed probability distributions or ranges of values based on experience and engineering judgement ([17]). Different uncertainty analysis techniques have been used in the past to study the structural reliability of cast iron pipes. In this paper, fuzzy numbers are used to represent uncertainties, since the fuzzy set theory is able to deal effectively with epistemic uncertainties that encompass subjectivity and vagueness and allows approximate reasoning. The theory also has the intrinsic ability to propagate uncertainties through the model. Uncertainty analysis in this paper is performed using fuzzy arithmetic (also referred to as possibilistic analysis, as this synergism was first proposed by Zadeh in 1978 [18]). Uncertainty in possibility theory is represented by dual measures, possibility denoted by $\Pi$ and necessity denoted by $N$. These dual
measures can be interpreted as plausibility and belief (certainty or surety) within the Dempster-Shafer [19] framework where evidence is nested. A complete discussion on how uncertainties using fuzzy set theory are incorporated in the pipe-soil interaction model is provided in [5].

Table 1 Corrosion parameters for two-phase model.

<table>
<thead>
<tr>
<th>Soil corrosivity</th>
<th>Minimum corrosion rate, (a) (mm/year)</th>
<th>(k) (mm)</th>
<th>(c) (1/year)</th>
<th>Maximum corrosion rate, (a + kc) (mm/year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very low (VLC)</td>
<td>0.0042</td>
<td>1.95</td>
<td>0.058</td>
<td>0.1173</td>
</tr>
<tr>
<td>Low (LC)</td>
<td>0.0210</td>
<td>9.75</td>
<td>0.058</td>
<td>0.5865</td>
</tr>
<tr>
<td>Moderate (MC)</td>
<td>0.0252</td>
<td>11.70</td>
<td>0.058</td>
<td>0.7038</td>
</tr>
<tr>
<td>High (HC)</td>
<td>0.0294</td>
<td>13.65</td>
<td>0.058</td>
<td>0.8211</td>
</tr>
<tr>
<td>Very high (VHC)</td>
<td>0.0336</td>
<td>15.60</td>
<td>0.058</td>
<td>0.9384</td>
</tr>
</tbody>
</table>

Fig. 1 Assumed pit corrosion rates for soils with different corrosivity.

Corrosion models together with information about soil corrosivity derived either from soil properties [20] or past experience (Table 1) can be used to estimate the growth rate of corrosion pits. The pit depth \(d_a\) obtained from NDT inspection (at current time, \(\tau_a\)) and known or assumed soil corrosivity can be used to back calculate the corrosion initiation time \(\tau_i\) using equation (5). Of course, the estimated time \(\tau_i\) cannot exceed pipe age, however if it does, it means that the assessed soil corrosivity is on the low side of what the current pit depth indicates and consequently, higher soil corrosivity has to be specified. Low soil corrosivity for the same pit depth will result in an earlier time for the initiation of corrosion as shown in Fig. 2.

Fig. 2 Schematics of pit growth in a very high and low corrosivity soils.

The solutions for each of the stress components (1) and (2) together with failure theory criterion as well as expression (5) that describes pit growth were evaluated using fuzzy arithmetic to obtain realistic ranges of values for factors of safety. The uncertainties in each of the data/parameters were expressed in terms of triangular fuzzy numbers (TFNs - lower and upper bounds and most likely values), based on available data, best engineering judgment and experience. ’Crisp’ (point) values were used for properties known with certainty.

Thus the time to failure \(\tau_f\) for a specific pipe with a known remaining wall thickness at the current time \(\tau_a\), and installed at time \(\tau_o\) can be obtained using the following time marching steps:

1. Estimate the time \(\tau_i\) a corrosion pit initiated using the pit depth \(d_a\) from NDT inspection and assess soil corrosivity.
2. Obtain pit depth \(d_\tau\) at time \(\tau\) from equation (5) for all \(\tau \geq \tau_a\).

3. Determine the fuzzy factor of safety \((FS)\) that corresponds to the pit depth \(d_\tau\) found in the previous step.

4. Repeat steps 2 and 3 for each time increment until the \(FS\) is below the critical factor of safety, i.e., \(FS < FS_{cr}\). The time when this occurs is defined as time to failure \((\tau_{cr})\).

6 SENSITIVITY ANALYSES

Two types of sensitivity analyses are described in this paper. The first type considers how the calculated fuzzy factors of safety \(FS_{de}\) decrease with growth of corrosion pit depth and the increase in unsupported bedding length in a 75 year old 150 mm (6") diameter cast iron main embedded in a moderately (Fig. 3a) corrosive environment. The pipe was assumed to have a single pit with a remaining wall thickness of 75\%\(t\) when inspected. It is also assumed that the leak and consequent loss of bedding initiate when the pit depth \(\geq 75\%t\) and increases to a maximum of five times pipe diameter \((D)\) when the pit has fully penetrated pipe wall thickness. At any given time, \(\tau\), a TFN (shaded area) in Fig. 3a shows in grey scale how the membership to \([minimum, most likely, maximum]\) fuzzy factors of safety changes with time as the pit depth grows, i.e., dark and light regions represent highest and lowest memberships of 1 and 0, respectively.

Possibility \((\Pi)\) and necessity \((N)\) measures for a fuzzy \(FS\) with respect to the ‘critical’ factor of safety, assumed here as \(FS_{cr} = 1\), were computed as explained in [5]. Fig. 3b shows how necessity, i.e., surety, of failure increases with time for various soil corrosivity levels. Time to failure ranges between 25 to 80 years for a 75 year-old cast iron main with a current pit depth of 75\%\(t\) and buried in very high (VHC) to moderately (MC) corrosive soils, respectively.

The second type of sensitivity analysis is not restricted to assessing the influence of specific variables but is broad in that it assesses the outcome of variability in any of the input data/parameters ([21]). This type of analysis also serves to identify the important uncertainties in order to prioritize additional data collection efforts. A Monte Carlo type simulation procedure with the rank correlation method was employed for the sensitivity analysis of the second type. This procedure was applied in the context of fuzzy input data generated randomly (1000 realisations) using the \(\alpha\)-cut concept of fuzzy sets. The \(\alpha\)-cut can be used to form a fuzzy confidence band, which can be viewed as a possibilistic confidence interval analogous to the probabilistic confidence interval. Details on implementing \(\alpha\)-cut based Monte Carlo simulation are provided in [5].

A sensitivity analysis was conducted on a cast iron pipe with the same characteristics as for the first analysis, except that the time frame considered was within 10 \(\pm\) 2.5 years following inspection. The sensitivity of \(FS\) to each of the input parameters is shown in the form of tornado graphs in Fig. 4 (distortional energy failure criteria). Tornado graphs facilitate the visualization of the positive or negative influences (% contribution) on the factor of safety, as a consequence of an increase in one specific input parameter. As expected, Fig. 4 shows that the factor of safety increases with an increase in fracture toughness \(K_{q}\), which contributes to the increase in pipe structural resistance. On the other hand, an increase of unsupported length \((b)\), and external loads \((q)\) leads to a reduction in the factor of safety. However, the maximum corrosion pitting rate \((a + kc)\) is the largest contributor towards the decrease in \(FS\).

The sensitivity analyses also show that some variables, e.g., minimum corrosion pitting rate \((a)\), soil modulus of elasticity \((E_s)\), internal \((p_i)\), temperature differential \((\Delta T)\) and transient pressure, have very low to insignificant influences on the structural factor of safety of small diameter buried pipes of these sizes.
Temperature differential ($\Delta T$) becomes a significant contributor to the decrease in $FS$ when the unsupported length ($b$) is small ($b = 1D$) but not when it is large, i.e., $b = 3D$. It can also be argued that some of these factors could be expressed as point estimates without compromising the accuracy of the results; consequently no effort should be invested in obtaining more precise values for these factors.

7 CONCLUSIONS

A previously developed pipe-soil interaction model was combined with failure theories and corrosion models to determine time to failure with uncertainties represented as triangular fuzzy numbers. The fuzzification of the mechanistic pipe-soil interaction and corrosion models combined with possibility theory provide a systematic approach to incorporating and propagating uncertainties and vagueness throughout the solution process. The possibilistic analysis also allows the designer/owner/operator to decide the level of risk that s/he is willing to take and thus define different repair and maintenance strategies at different risk tolerances.

The sensitivity analyses clearly show that the long-term performance of buried cast iron is dictated by pit growth rate, unsupported length, fracture toughness and temperature differential. These sensitivity analyses strongly suggest that reducing pit depth (graphitization) growth by using effective corrosion control can be an effective way to decelerate breakage growth rate. This observation corroborates the experience of utilities and corrosion engineers. The proposed methodology is a viable approach to practically determining time to failure of cast iron mains under different soil conditions.

References


