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EQUATIONS FOR SOLAR HEAT GAIN THROUGH WINDOWS

BY

D. G. STEPHENSON

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EQUATIONS POUR CALCULER LES GAINS DE CHALEUR SOLAIRE AU TRAVERS DES FENETRES

SOMMAIRE

Une analyse des données de radiation solaire obtenues à Scarborough, en Ontario, montre qu'au Canada l'insolation peut être nettement supérieure aux valeurs données par les courbes standard de radiation solaire de Moon. On peut représenter les données solaires par une seule formule analytique comprenant un coefficient d'absorption atmosphérique et une valeur apparente de la constante solaire. Cette formule permet de calculer l'insolation de n'importe quelle surface et de déterminer à quel moment et à quelle date peut se produire l'insolation maximum. Des formules simples établissent le rapport qui existe entre les gains de chaleur solaire au travers des fenêtres et l'heure, la date, la latitude, l'orientation des bâtiments, le type des vitres et les stores s'il y en a. Ces formules peuvent être employées pour programmer une calculatrice électronique dans le but d'obtenir des valeurs estimées des gains instantanés de chaleur au travers des fenêtres d'un bâtiment.

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Equations for Solar Heat Gain Through Windows

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An analysis of solar-radiation records obtained at Searborough, Ontario, indicates that the insolation in Canada can be significantly greater than the values given by Moon's[1] standard solar-radiation curves. It is possible to represent the solar data by a single analytical expression involving an atmospheric extinction coefficient and an apparent value of the solar constant. This expression allows the calculation of insolation on any surface and the determination of the time and date when the maximum insolation can occur for any surface. Simple expressions relate the time, date, latitude, building orientation, and the type of window glass and shading with the solar heat gain through windows. These can be used to program any digital computer to compute design values of the instantaneous heat gain through the windows of a building.

SOLAR radiation should be considered at a very early stage in the design of a building. It can influence such basic decisions as: orientation, the amount and type of glass to be used in the various facades, whether it should be shaded, and whether the building will have to be air-conditioned. Ideally, the architect should have at his disposal comparative values of solar heat gain for all the wall-window combinations he is considering for any particular project. The calculation of solar heat gain can be programmed on even a very modest digital computer; and once a program is available it is practical to use it for the preliminary design analysis as well as for the detailed design of the cooling system.

This paper presents a system of equations relating the solar heat gain through windows to the basic data and gives values for the data. It is a relatively simple task to organize this information so that it can be used by any available computer. The equations are separated into three categories relating to: (1) Solar irradiation; (2) The angles at which the solar beam strikes any surface, and (3) The transmission and absorption factors for windows.

Each of these subjects is discussed separately before the three are combined to indicate the maximum value of solar heat gain and when it can occur. The geometric relationships between wall orientation and the position of the sun are certainly not new. They are, however, cast in a form that is convenient for computer programming.

This paper is a contribution from the Division of Building Research, National Research Council, Canada, and is published with the approval of the Director of the Division. It was presented at a Regional Technical Conference of the Engineering Institute of Canada at Kingston, Ontario, on 9 May 1964, but has not been previously published.

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Basic Equations for Solar Irradiation

Design values of the solar radiation reaching the surface of the earth should be based on a statistical analysis of solar-radiation records. If they are not available, the only alternative is to calculate the values that would obtain for some assumed atmosphere. In 1940 Moon[1] followed this latter course and published his proposed standard solar-radiation curves for engineering use. These data have been used since that time for most engineering design calculations involving solar energy. In the meantime, there has been a great increase in the number of stations making continuous measurements of solar radiation, so that it has become possible to check Moon's theoretical standard against measured values.

Ten years ago Parmelee[2] published results that showed observed values exceeding Moon's values by about 10 percent. In 1958 Threlkeld and Jordan[3] made a comparison between observed values of solar radiation on clear days and values indicated by an analysis similar to Moon's. They found that the observed values at three widely separated locations in the United States were in general agreement with one another, but higher than Moon's standard. They, therefore, defined a new standard. In Threlkeld's papers the term, clear day, is taken to mean an average cloudless day rather than a design day. In fact, in 1963 Threlkeld[4] reported that on some days the radiation at Minneapolis exceeded his standard by as much as 20 percent.

Aside from the fact that the various proposed standards appear to be too low for normal design purposes, they are not well suited for inclusion in a computer program. A new analysis has been made, therefore, with the two-fold objective of determining design values of solar irradiation that are applicable for building design in Canada and of obtaining the data in a form that is convenient for computer programming.

The solar radiation incident on the outside surface of a building consists of three components: the direct solar beam, I_D; the scattered solar radiation coming from all parts of the sky, I_S; and the solar radiation reflected onto the surface by neighbouring surfaces, I_R. These components will be discussed in turn.

Radiation Coming Directly from the Sun

The Meteorological Branch of the Department of Transport, Canada, publishes the measured values of

TABLE OF NOMENCLATURE

A	= area of a surface
I_D	= direct solar insolation on any surface
I_S	= sky-scattered solar insolation on any surface
I_R	= reflected solar radiation reaching any surface
I	= total solar insolation on any surface = $I_D + I_S$
S	= total solar irradiation on any surface = $I_D + I_S + I_R$
ρ	= reflectivity of a surface
a further subscript on any of the above quantities identifies the surface	
a	= atmospheric extinction coefficient
B	= $\frac{(I_s)_{\text{Hor.}}}{DNI}$
C_n	= polynomial coefficients for absorption and transmission factors of windows
DNI	= direct normal insolation at ground level
$F_{i\to j}$	= the geometric view factor between surfaces i and j; i.e. the fraction of the radiation from surface i that falls on j
$F(\cos\theta)$	= $\frac{(I_s)_{\text{Ver.}}}{(I_s)_{\text{Hor.}}}$, a function of incident angle
I^*	= apparent solar constant
M	= secant of solar zenith angle (approximately equal to air mass)
α, β, γ	= the angles between the solar beam and a horizontal axis pointing south, a horizontal axis pointing west and a vertical axis respectively; γ being the zenith angle
l, m, n	= direction cosines of the normal to a surface referred to the same axis system as used for solar angles
θ	= angle of incidence of solar beam on a surface $\cos\theta = l\cos\alpha + m\cos\beta + n\cos\gamma$
ψ	= latitude angle
δ	= solar declination angle
h	= solar hour angle
Az	= solar azimuth angle
η	= wall solar azimuth
φ	= profile angle
superscript′	= indicates value at time of maximum insolation

solar radiation falling on horizontal surfaces at about 20 stations across Canada. These values are the sum of the radiation coming directly from the sun and the radiation reaching the surface of the earth after being scattered by the atmosphere. These data alone cannot be used to calculate the insolation on surfaces that are not horizontal; they must be complemented by a simultaneous measurement of the direct solar beam. As the direct normal insolation, DNI, is measured at only one station in Canada, the National Radiation Centre at Scarborough, Ontario, only the Scarborough records are of use in this study. These direct normal measurements were begun in June 1960 and have been available since that time (with a gap of a few months in 1961 when the pyrheliometer was damaged by lightning). The published values are averages over a one-hour period, ending on the hour in local apparent time. In the correlation that follows it is assumed that these hourly averages are the same as the instantaneous values at the mid-point of the averaging period.

About 20 of the clearest days in the period between June 1960 and June 1963 were selected for analysis. The criteria for selection were: (1) a high value at midday, (2) symmetrical value for the morning and afternoon, and (3) low values of the sky radiation as measured by a shaded pyranometer. The days that were selected represent all seasons, but there are more samples per month for summer and autumn than for winter and spring. This may be a result of using only a three-year period or it may indicate that the frequency of clear days is higher in summer and autumn.

For each of the selected days the zenith angle* was calculated for every hour on the half hour (local apparent time). The logarithm of DNI was plotted vs the secant of the zenith angle (i.e. air mass). The curves in Fig. 2 are typical of these plots. The data points fall quite close to a straight line so that DNI can be represented by

$$DNI = I^* e^{-a/\cos\gamma}$$

where $\log I^*$ is the ordinate of the intersection of the line through the data points with the axis $M = 0$. The value of a is found from the expression

$$a = \ln(I^*/I_3)/3$$

where $\log I_3$ is the ordinate of the line through the data points at $M = 3$.

Moon's standard curve is shown on Fig. 2 for comparison. It is lower than any of the days selected for this study.

The values of a vary during the year, as is shown in Fig. 3. In winter they are only half the summer value. The lower the value of a the higher DNI is for any specified air mass; thus, design values of DNI should be based on the minimum values of a that can occur at any date. A period of observation of only three years is too short to establish a minimum for each month on any statistical basis, but the curve drawn through the points on Fig. 3 is a suggested basis for calculating solar irradiation on very clear days. It may need to be modified slightly when more data are available.

Short-Wave Radiation from the Sky

The diffuse or sky-scattered short-wave radiation incident on a horizontal surface was deduced from the published value of DNI and $I_{\text{Hor.}}$. The ratio I_H/DNI was plotted vs $\cos\gamma$ for all the days included in this

* The angles used in this paper are defined on the diagram in Fig. 1.

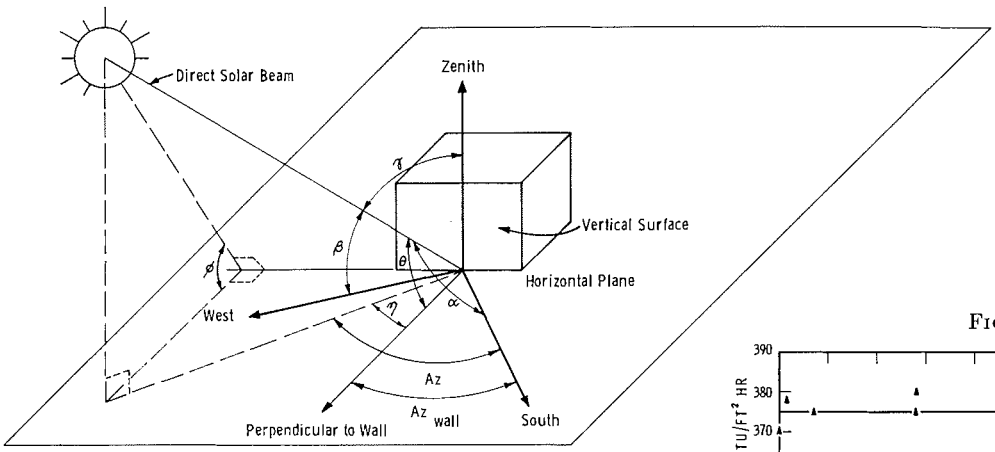


FIG. 1—Definition of solar angles.

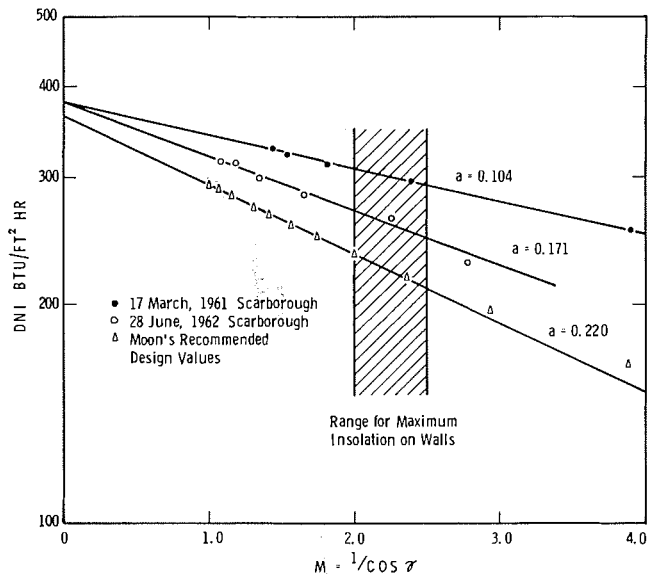
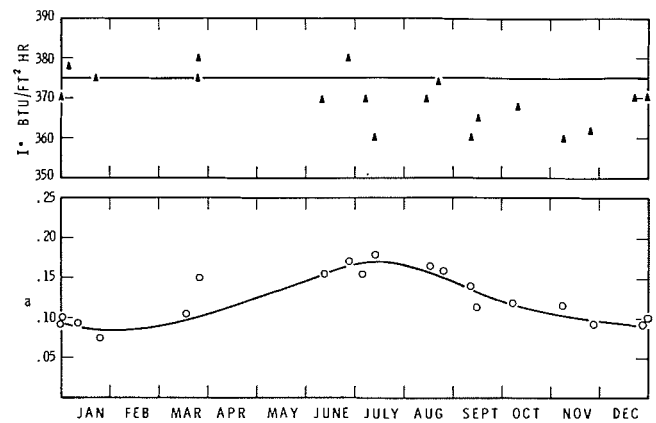


FIG. 2—Direct normal irradiation vs air mass for clear days at Scarborough.

FIG. 3—Annual variation of I^* and a.



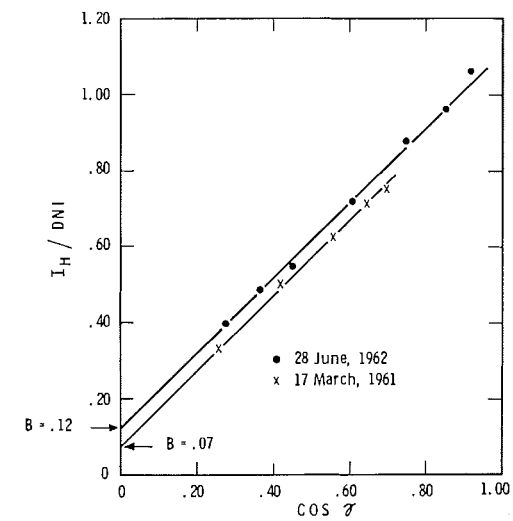


FIG. 4—Variation of I_H/DNI vs $\cos\gamma$.

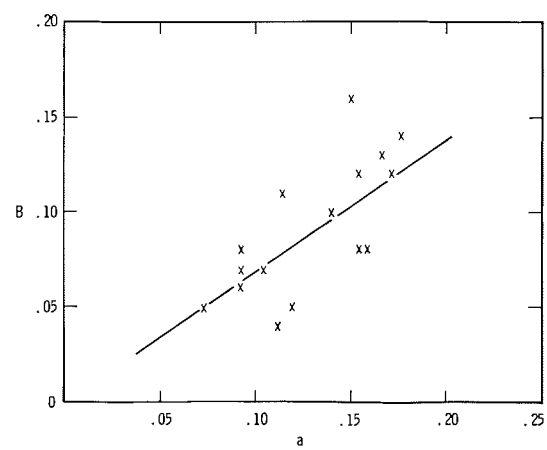


FIG. 5—Scattering vs extinction coefficient.

study. Figure 4 is this type of plot of the data for the days shown in Fig. 2. The data points lie very close to a straight line with unit slope, so that

$$I_H = DNI \cos\gamma + DNI\, B$$

The first term on the right side of this equation is the component of I_H that is due to the direct solar beam, since γ is the incident angle of the solar beam on a horizontal surface; and the second term is the sky component. The records for very clear days show that the sky-scattering is directly proportional to DNI and that the proportionality constant B can be related to the atmospheric extinction coefficient a. The values of B are plotted in Fig. 5 vs the corresponding values of a. This simply indicates that the atmospheric extinction coefficient increases when there is an increase in scattering by the atmosphere. The spread of the data points in Fig. 5 is partly the result of errors in the graphical determination of B, but it also reflects the fact that a depends on absorption of radiation by CO_2 and H_2O in the atmosphere while B is not affected by absorption.

TABLE 2—POLYNOMIAL COEFFICIENTS FOR ABSORPTION AND TRANSMISSION FACTORS OF WINDOWS

	Ordinary Window Glass		Heat-Absorbing Plate		Double Glazing Ordinary Window Glass		
	Absorption	Transmission	Absorption	Transmission	Absorption Outer Pane	Absorption Inner Pane	Transmission
C_0	0.01154	−0.00885	0.01406	−0.00835	0.01407	0.00228	−0.00401
C_1	0.77674	2.71235	4.15958	0.92766	1.06226	0.34559	0.74050
C_2	−3.94657	−0.62062	−15.06279	2.15721	−5.59131	−1.19908	7.20350
C_3	8.57881	−7.07329	27.18492	−8.71429	12.15034	2.22366	−20.11763
C_4	−8.38135	9.75995	−23.88518	9.87152	−11.78092	−2.05287	19.68824
C_5	3.01188	−3.89922	8.03650	−3.73328	4.20070	0.72376	−6.74585
Diffuse values	0.0544	0.7990	0.4641	0.4268	0.0600	0.0451	0.6797

NOTES: 1) Heat-absorbing plate has a transmission for normal incidence = 50 percent.

2) Diffuse values $= 2\left[\frac{C_0}{2} + \frac{C_1}{3} + \frac{C_2}{4} + \frac{C_3}{5} + \frac{C_4}{6} + \frac{C_5}{7}\right]$.

3) Value for incident angle of $\theta = \sum_{n=0}^{5} C_n \cos^n \theta$.

This method of arriving at the sky-scattered shortwave radiation incident on a horizontal surface includes the radiation from all the sky except the very small solid angle that comprises the field of view of the pyrheliometer. The forward-scattered radiation in this region of the sky around the sun is included with the direct solar beam. The values for the sky-scattered component evaluated in this way are in general agreement with those measured by a shaded pyranometer. The published values of the shaded pyranometer results have been adjusted, however, to compensate for the segment of the sky obscured by the shade ring. This correction requires some assumption about the variation in intensity of the scattered radiation over the sky. No such assumption is involved in the values for the scattered radiation given by the method used in this analysis.

Threlkeld[4] has shown that the diffuse shortwave radiation from the sky incident on a vertical surface is related to the concomitant value for a horizontal surface and the angle of incidence of the direct beam on the vertical surface.

Combining this with the observation that

$$I_{S,\text{Hor.}} = B \times DNI$$

gives

$$I_{S,\text{Ver.}} = DNI \times B \times F\{\cos\theta\}$$

where $F\{\cos\theta\}$ is the empirical function given in Threlkeld's paper. It can be well represented by the following expression: when

$$\cos\theta > -0.2$$

$$F\{\cos\theta\} = 0.55 + 0.437\cos\theta + 0.313\cos^2\theta$$

and when $\cos\theta < -0.2$

$$F\{\cos\theta\} = 0.45$$

The total insolation, I, on a vertical surface is given by

$$I = I^* e^{-a/\cos\gamma}(\cos\theta + B \times F\{\cos\theta\})$$

It is shown in the section dealing with the equations for solar position that the values of $\cos\theta$ and $\cos\gamma$ can be calculated quite easily, and that the insolation on any wall can be determined by the above equation when the values of I^*, a and B are specified. Tentative design values of these data for the southern part of Canada are given in Figs. 3 and 5.

Irradiation of Building Surfaces

The surface of a building can also receive significant amounts of solar radiation by reflection from adjacent surfaces.

If the surfaces reflect diffusely the total irradiation, S, falling on any surface can be expressed as

$$A_1S_1 = A_1I_1 + A_2S_2\rho_2F_{2\to1} + A_3S_3\rho_3F_{3\to1} + \cdots$$

$$A_2S_2 = A_1S_1\rho_1F_{1\to2} + A_2I_2 + A_3S_3\rho_3F_{3\to2} + \cdots \text{ etc.}$$

As $A_1F_{1\to2} = A_2F_{2\to1}$ etc. the areas can be divided out of these equations to give

$$S_1 = I_1 + \rho_2F_{1\to2}S_2 + \rho_3F_{1\to3}S_3 + \cdots$$

$$S_N = \rho_1F_{N\to1}S_1 + \rho_2F_{N\to2}S_2 + \rho_3F_{N\to3}S_3 + \cdots + I_N$$

Strictly speaking, this set of equations should be solved simultaneously to yield the values of S when the values of I and the geometric view factors are known. It is sufficiently accurate for most practical purposes, however, to use the values of the insolation I instead of the corresponding values of S in the expressions on the right side of these equations.

Equations for Position of the Sun

The angles used to describe the direction of the solar beam relative to a building surface are shown in Fig. 1. The basic coordinate axes are lines pointing south, west and to the zenith. The incident angle, θ, is the angle between the solar beam and a line perpendicular to the wall. The value of $\cos\theta$ can be determined quite readily from the following expressions:

$$\cos\theta = l\cos\alpha + m\cos\beta + n\cos\gamma$$

The direction cosines of the solar beam are functions of the latitude ψ, the hour angle h, and the date (i.e. the value of the solar declination, δ).

$$\cos\gamma = \sin\psi \times \sin\delta + \cos\psi \times \cos\delta \times \cos h$$

$$\cos\beta = \cos\delta \times \sin h$$

$$\cos\alpha = \pm(1 - \cos^2\beta - \cos^2\gamma)^{\frac{1}{2}}$$

$\cos\alpha$ is positive when $\cos h \tan\delta/\tan\psi$.

The solar declination varies from +23.5 degrees at the summer solstice, through zero at the equinox, to −23.5 at the winter solstice. The values at intermediate dates are given in Table 1. The solar azimuth angle, Az, is related to the direction cosines of the solar beam by the simple expression

$$\tan Az = \cos\beta/\cos\alpha$$

or

$$\sin Az = \cos\beta/\sin\gamma;$$

the wall solar azimuth, η, which is the difference between the wall azimuth and the solar azimuth, is given by

$$\cos\eta = \cos\theta/\sin\gamma$$

Area of Glass Receiving Direct Sunlight

The area of a window that is shaded from direct solar radiation by the window frame or by an external shading fin can be computed quite simply. The width of the shadow cast by a projection from the edge of a window is $L \times \tan\eta$, where L is the perpendicular distance from the plane of the glass to the outermost edge of the projection. Tan η is related to the incident and zenith angles by the equation

$$\tan\eta = \frac{(1 - \cos^2\gamma - \cos^2\theta)^{\frac{1}{2}}}{\cos\theta}$$

When $\cos\theta$ is negative the surface receives no direct sunshine. For a window with identical shades at both sides, the sign of the square root is unimportant, since the change in sign indicates only that the shadow has changed from the left to the right side of the window.

The relevant solar angle for a horizontal projection from the top of a window is the profile angle φ. It is related to the incident and zenith angles by the simple expression

$$\tan\varphi = \cos\gamma/\cos\theta$$

The height of the shadow cast by a horizontal projection extending a distance L from the plane of the window is $L \times \tan\varphi$. The sunlit area of a window is, therefore, (height $- L\tan\varphi$) $\times$ (width $- L\tan\eta$).

Transmission and Absorption Factors for Glass

The absorptivity and transmissivity of a window depend on the type and thickness of the glass; on whether it is single or double glazed; on the angle that the incident beam of light makes with the normal to the surface; and on the degree of polarization of the incident beam. Values of absorptivity and transmissivity have been computed and tabulated as functions of incident angle for single- and double-glazed windows by Mitalas and Stephenson.[5] These data are fine for hand computation, but are not well suited for inclusion in a computer program. For this purpose a polynomial in $\cos\theta$ has been fitted to the data,

i.e.

$$T(\theta) = \sum_{n=0}^{5} C_n \times \cos^n\theta$$

TABLE 1—SOLAR DECLINATION VS CALENDAR DATE

Declination (degrees)	Dates (from 1964 Nautical Almanac)	
23.5	21 June	
20	24 July	20 May
15	12 August	1 May
10	27 August	16 April
5	10 September	2 April
0	23 September	21 March
−5	6 October	8 March
−10	19 October	24 February
−15	3 November	9 February
−20	22 November	22 January
−23.5	22 December	

Values of the coefficients for these polynomials will be published shortly by the National Research Council. Table 2 presents the coefficients for a single sheet of ordinary window glass, a single sheet of heat absorbing glass (that transmits about 50 percent at normal incidence) and a double window with both panes of ordinary glass.

Cosine θ has been used as the argument for these polynomials for two reasons: it is evaluated directly from the direction cosines of the solar beam and the normal to the surface, so that no inverse trigonometric function needs to be evaluated; and the integration t obtain the values for diffusely incident radiation is particularly simple to evaluate:

$$T_{\text{Diffuse}} = \int_0^{\pi/2} T(\theta)\sin 2\theta\, d\theta = 2\sum_{n=0}^{5}\frac{C_n}{n+2}$$

The values for diffuse radiation, obtained from the coefficients in this way, are included in Table 2.

Maximum Insolation on a Vertical Surface

The incident angle of the solar beam on any vertical surface can be expressed as

$$\cos\theta = \cos\eta \times \sin\gamma$$

Thus, the direct insolation on a vertical surface is

$$I_{D,\text{Ver.}} = I^* e^{-a/\cos\gamma} \times \cos\eta \times \sin\gamma$$

This is maximum when η is zero and γ satisfies the relationship

$$\cos\gamma = a \times \tan^2\gamma$$

The values of the angles that obtain when the insolation is maximum are designated by a prime superscript. Values of γ' corresponding to the values of a that occur on very clear days at Scarborough are shown in Fig. 6. The range is only from 60 degrees in summer to about 67 degrees in winter.

The time and date when the maximum insolation can occur are functions of latitude and orientation.

$$\cos\beta' = \sin Az_{\text{Wall}} \times \sin\gamma'$$

Combining this with the equation for $\cos\gamma$ gives

$$\cos\gamma' = \sin\psi\sin\delta' + \cos\psi(\cos^2\delta' - \cos^2\beta')^{\frac{1}{2}}$$

which can be solved for the declination δ'. The corresponding hour angle is given by $\sin h' = \cos\beta'/\cos\delta'$.

West and east walls have wall azimuths of + and −90 degrees, respectively; the maximum insolation on these facades therefore occurs when $\tan Az = \infty$, i.e. $\cos \alpha = 0$. This obtains when $\cos h = \tan \delta / \tan \psi$, and when this is substituted into the expression for $\cos \gamma$ it gives

$$\sin \delta' = \cos \gamma' \times \sin \psi$$

For example, if $\psi = 45$ degrees north and $a = 0.17$, ($\gamma' = 60$ degrees), the requisite declination for maximum insolation is +20.7 degrees, which corresponds to about 21 July and 17 May. The hour angle at which this maximum occurs is ±68 degrees (plus for a west wall and minus for an east wall), corresponding to $4\frac{1}{2}$ hours from solar noon.

A south-facing wall has $Az_{\text{Wall}} = 0$ so that $\cos \beta' = 0$. Therefore,

$$\cos \gamma' = \sin \psi \sin \delta' + \cos \psi \cos \delta'$$
$$= \cos (\psi - \delta')$$

and $\sin h' = 0$

so that $\delta' = \psi - \gamma'$

and $h' = 0$

As γ' is greater than the latitude for all but the far north of Canada, the maximum will occur when the solar declination is negative, i.e. in autumn and winter when a is near its lowest value and γ' near its maximum. For a location at 45 degrees north latitude, assuming $\gamma' = 65$ degrees, the value of δ' is −20 degrees, which corresponds to 21 November and 21 January.

The reflection from an adjacent light-coloured surface can increase the irradiation on a vertical surface quite significantly. Fresh snow has a very high reflectivity and much of the reflected energy is incident on vertical surfaces at an angle close to the incident angle for the direct solar beam. The total irradiation on a south facing wall at Ottawa ($\psi = 45$ degrees) has been measured for the past two years. These records show that the irradiation can be as high as 350 Btu per sq ft hr at noon on any clear winter day following a snowfall.

Significance of These Data

The maximum insolation on east and west walls has been shown to occur about the 21st of July for the latitudes of southern Canada when the extinction coefficient for the atmosphere is about 0.17. For these conditions the direct normal solar beam, *DNI*, is 267 Btu per sq ft hr and the direct beam falling on the wall is 231 Btu per sq ft hr. The diffuse radiation from the sky that is incident on the wall adds another 12 percent of *DNI* (i.e. 32 Btu per sq ft hr), and radiation reaching the wall surface after reflection from the ground adds between 10 and 15 Btu per sq ft hr. Thus the total diffuse irradiation is about 45 Btu per sq ft hr.

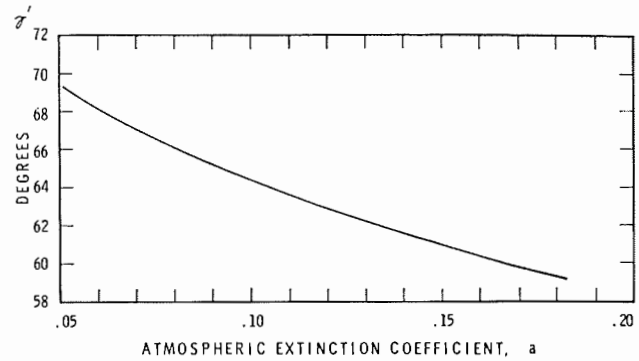


FIG. 6—Zenith angle for maximum insolation on a vertical surface vs atmospheric extinction coefficient.

A single sheet of ordinary window glass transmits 87 percent and absorbs 5.1 percent of the direct solar irradiation when $\theta = 30$ degrees (for diffuse irradiation the transmission is 80 percent and the absorption 5.4 percent). Thus, the maximum transmission through a single glazed window in a west wall is 237 Btu per sq ft hr, and the glass absorbs 14.2 Btu per sq ft hr. If the convection conditions are similar on the two sides of the glass, half of the absorbed energy is transferred to the inside of the building, making 244 Btu per sq ft hr the total instantaneous heat gain from solar radiation through a single sheet of ordinary window glass in a west wall. This is 22 percent higher than the maximum value given in the widely used ASHRAE Guide and Data Book[6].

Acknowledgment

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