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Analytical method for designing gradient-index fiber probes

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Abstract. An analytical method was investigated to design the gradient-index (GRIN) fiber probe based on characteristic parameters of a Gaussian beam propagating through the probe. First, a typical model of the GRIN fiber probe was presented, consisting of a single mode fiber, a no-core fiber (NCF), and a GRIN fiber lens. Second, a complex beam parameter matrix transformation method was adopted to derive the mathematical expressions of characteristic parameters, such as beam waist location, beam waist radius, and Rayleigh range. Then, MATLAB software was used to analyze the impact of the length of NCF and the length of the GRIN fiber lens on the characteristic parameters. Finally, performance comparison was performed between the calculation results and the experimental data published previously. The calculation results are in agreement with the experimental data and thus validate the presented analytical method for designing GRIN fiber probes. In addition, the characteristic parameters of a Gaussian beam going through the GRIN fiber probe will stay the same when the length of the GRIN fiber lens increases every 1/2 pitch length, which could be used to decrease the fabrication complication of GRIN fiber probes by increasing the length of GRIN fiber lens periodically.

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1 Introduction

Optical coherence tomography (OCT) is a noninvasive, noncontact, depth-resolved imaging technique with high-resolution and high sensitivity.¹ With rapid improvement of acquisition speed and axial resolution of OCT over recent years, OCT is becoming increasingly promising for a variety of diagnostic and surgical applications. In an OCT system, an optical probe transmits and focuses a light beam (e.g., Gaussian beam) inside a sample of interest, then collects the backreflected light carrying information about the sample and sends it to a signal processing system. Therefore, the optical probe is a key part of the OCT system, whose focusing performance plays an important role in the imaging quality of OCT. For example, the beam shape, e.g., waist location, waist size, and Rayleigh range, could have a great impact on the penetration depth, lateral resolution, and depth of field of OCT imaging.

Miniaturization of the optical imaging probe and the OCT system is a critical topic in the enhancement of OCT for *in vivo* human and animal studies, and has been investigated by researchers in recent years. Among the miniaturized probes, a gradient-index (GRIN) lens is promising since it is easy to be integrated into an endoscopic environment. For example, Guo and Xie et al. designed the GRIN lens rod-based probes for endoscopic OCT.²⁻⁴ Singh et al. studied a miniature catheter OCT probe with a scanning micromirror device.⁵ Aljaseem et al. developed a miniaturized fiber optical tunable endoscopic probe with a diameter of ~ 4 mm.⁶

Meemon et al. investigated optical design of a dynamic focus catheter for high-resolution endoscopic OCT.⁷ Min et al. proposed a fiber-based hand-held scanning probe suitable for the sample arm of OCT.⁸ In addition, Jung et al. reported the numerical analysis of gradient-index lens-based optical probes.⁹ The diameter of the optical probes investigated by the researchers above is typically at the level of several millimeters, which may be suitable for *in vivo* OCT imaging in mucosal layers gastrointestinal tract, intra-arterial, and intravascular, etc. However, for the imaging of small lumen, the narrow space in the deep tissues and organs (e.g., cardiovascular) of human beings and small animals, a key concern is the possible damage from the mechanical insertion of the optical probe. Therefore, it is critical to develop an ultra-small optical probe that is compatible with current optical biomedical imaging systems.

The GRIN fiber probe is defined as an all-fiber-type ultra-small optical probe for miniaturization of the OCT system, typically consisting of a single-mode fiber (SMF), a no-core fiber (NCF), and a GRIN fiber lens (see Fig. 1 below). In 2002, Swanson et al. proposed the design of GRIN fiber lens-based miniature optical probes.¹⁰ Then, Reed and Jafri and Li et al. respectively demonstrated the usage of such probes for OCT imaging system.¹¹⁻¹³ However, all of them did not present the design methods for the probes theoretically. Since 2007, Dr. Mao studied the fabrication and performance testing methods of such GRIN fiber probe.^{14,15} However, the length of the designed GRIN fiber lens is typically shorter than 0.2 mm, which obviously requires high precision cutting of fiber length and thus high grade fabrication of the ultrasmall optical probes. Therefore, a thorough theoretical

method is required to identify the function of each component and obtain optimal probe design parameters. However, the design theory of the GRIN fiber probe has been largely overlooked in previously published papers.

In this paper, in order to investigate the design parameters of the GRIN fiber probe more comprehensively, the complex beam parameter matrix transformation method was adopted to derive the mathematical expressions of characteristic parameters, e.g., waist location, waist radius, and Rayleigh range of a Gaussian beam propagating through the probe. MATLAB software was used to analyze how the length of NCF and the length of a GRIN fiber lens impact the characteristic parameters. In addition, a focusing property comparison was performed between the calculation results and experimental data published previously. As a result, this paper provides a fundamental theory and an effective analytical method for designing GRIN fiber probes.

2 Model of GRIN Fiber Probe

Figure 1 shows a typical model of a GRIN fiber probe, which is composed of a SMF, a NCF, a GRIN fiber lens, and an air path. The SMF is connected with the detection arm of the OCT system, and guides the light beam into the NCF. The use of an NCF is able to improve the working distance of the probe by expanding the beam and thus can overcome the problem of limited mode field diameter of the SMF. However, the NCF may also reduce the lateral resolution of OCT imaging due to increasing the size of the focus spot. Therefore, the length of the NCF should be chosen appropriately. On the one hand, when the NCF is too long, there may exist overflowing of some light energy off the probe side, which will finally reduce the beam coupling efficiency. On the other hand, a too short NCF may result in the failure of an expanding beam and require high fabrication compliance of the probe.

The GRIN fiber lens is the most crucial component of the probe model. It has the ability of self-focusing due to a continuous change of refractive index within the lens material, and can be easily integrated with other planar optical components (e.g., SMF and NCF) by means of fusing or gluing. It should be noted that, in order to ensure minimum back reflection, the indices of the center of the GRIN fiber lens and NCF and the core of the SMF should be as close as possible. In the ideal case, the radial refractive index profile of a GRIN fiber lens is nearly parabolic shape, which can be

Fig. 2 Schematic diagram of a GRIN fiber lens imaging with different pitch lengths.

approximately expressed as,

$$n(r) = n_0 \left(1 - \frac{g^2}{2} r^2 \right), \quad (1)$$

where n_0 represents the refractive index at the center of the profile, r is the radius, and g is the gradient constant.

As shown in Fig. 2, the GRIN fiber lens has a periodicity of length for the imaging of an objective AB, which can be described by a parameter named pitch. The period or pitch length P_L is written as,

$$P_L = 2\pi/g. \quad (2)$$

By judging from Fig. 2 and formula (2), the pitch length of the GRIN fiber lens does not depend on the entrance height and angle of light rays, but its gradient constant g only. The focusing performance of a GRIN fiber lens can be analyzed given the relative parameters of light source and the lens itself. As shown in Fig. 2, the image of a point source on the entrance surface through a 1/4-pitch length of lens is an infinite or collimated beam, and a collimated beam becomes a point source correspondingly. For a 1/2-pitch length of lens, there is an inverted image on the exit surface of the lens. While for a 1- (2, 3, or more, respectively) pitch length of lens, there is an image on the exit surface identical to the object. In the case of the design of a GRIN fiber probe for OCT imaging, the length of the GRIN fiber lens should be close to a 1/4-pitch in order to obtain optimum imaging quality. However, such length of a GRIN fiber lens will have a limited focal length due to its strong focusing performance. In addition, too short of a length of the GRIN fiber lens may bring trouble in fiber cutting during the fabrication process of the GRIN fiber probe. Therefore, several factors should be considered for the choice of the length of a GRIN fiber lens.

Fig. 1 Model of a GRIN fiber probe with the light beam profile in red lines. (Color online only.)

3 Mathematical Expressions of Characteristic Parameters

In order to study the optical performance of the GRIN fiber probe more comprehensively, the complex beam parameter matrix transformation method is adopted to derive the mathematical expressions of characteristic parameters, e.g., waist location, waist radius, and Rayleigh range of a Gaussian beam propagating through a GRIN fiber probe. As shown in Fig. 1, an approximate Gaussian beam is output from the SMF with a wavelength of λ and a radius of ω_0 , into the NCF with an index of n_1 , and then into the GRIN fiber lens with an index profile described by formula (1). The light beam is eventually focused into a spot with a waist radius of W and a waist location of Z_w in air. The refractive index of air is n_2 . The refractive index at the center of the GRIN fiber lens is n_0 . L_0 and L represent the length of the NCF and the length of the GRIN fiber lens, respectively. Planes 1 to 6 denote the input plane, two interfaces between the NCF and the GRIN fiber lens, two interfaces between the GRIN fiber lens and the air, and the focal plane, respectively.

From the method of ray matrix transformation of the complex beam parameter, the complex beam parameter $q(z)$ is given by Eq. (3):¹⁶

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i \frac{\lambda}{n\pi\omega^2}, \quad (3)$$

where $R(z)$ is the radius of curvature of a Gaussian beam, ω is the beam waist, λ is the free-space wavelength of the Gaussian beam, n is the refractive index of the medium, and z is the distance of the beam propagating from the input plane. The transformation of the complex beam parameter $q(z)$ from an input plane q_1 to an output plane q_2 is represented below using the ABCD matrix method,

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D}, \quad (4)$$

where $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is the transformation matrix between input plane and plane of interest. The input plane is the interface between the SMF and the NCF. By inputting the beam waist conditions into Eq. (3), i.e., $R(z) \rightarrow \infty$ and $\omega = \omega_0$, we can get the following expression:

$$\frac{1}{q_1} = -i \frac{\lambda}{n_1\pi\omega_0^2}. \quad (5)$$

Setting $m = \lambda/n_1\pi\omega_0^2$, we can obtain the formulas for the plane of interest below,

$$\frac{1}{R(z)} = \frac{AC + m^2BD}{A^2 + m^2B^2}, \quad (6)$$

$$\omega(z) = \omega_0 \sqrt{\frac{n_1}{n_2} \left(\frac{A^2 + m^2B^2}{AD - BC} \right)}. \quad (7)$$

When the plane of interest is the focal plane, there is a beam waist condition of $R(z) \rightarrow \infty$, and we can get the following equation,

$$AC + m^2BD = 0. \quad (8)$$

At the input plane, the beam waist radius ω is ω_0 . The focal plane is behind the output plane in the air. According to Ref. 9, the transformation matrix between the input plane and the focal plane can be expressed as:

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = M_{56}M_{45}M_{34}M_{23}M_{12}, \quad (9)$$

where M_{ij} denotes the transformation matrix between plane $i = (1, 2, 3, 4, 5)$ and plane $j = (2, 3, 4, 5, 6)$.

$$M_{12} = \begin{bmatrix} 1 & L_0 \\ 0 & 1 \end{bmatrix}, \quad M_{23} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_0} \end{bmatrix},$$

$$M_{34} = \begin{bmatrix} \cos(gL) & \frac{1}{g} \sin(gL) \\ -g \sin(gL) & \cos(gL) \end{bmatrix},$$

$$M_{45} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_0}{n_2} \end{bmatrix}, \quad M_{56} = \begin{bmatrix} 1 & z_w \\ 0 & 1 \end{bmatrix}.$$

Here, z_w denotes the waist location. The following expressions of components of the matrix M can be derived from Eq. (9),

$$\begin{cases} A = \cos(gL) - \frac{n_0}{n_2} z_w g \sin(gL) \\ B = \left(L_0 + \frac{n_1 z_w}{n_2} \right) \cos(gL) \\ \quad + \left(\frac{n_1}{n_0 g} - \frac{n_0 L_0 z_w g}{n_2} \right) \sin(gL) \\ C = -\frac{n_0 g}{n_2} \sin(gL) \\ D = -\frac{n_0 g L_0}{n_2} \sin(gL) + \frac{n_1}{n_2} \cos(gL). \end{cases}$$

Then, the expression of the beam waist location z_w can be solved from Eq. (8),

$$z_w = \frac{n_1 n_2 L_0 m^2 \alpha^2 - n_1 n_2 L_0 m^2 \beta^2 + \left(n_0 n_2 g + n_0 n_2 g L_0^2 m^2 - \frac{n_2 n_1^2 m^2}{n_0 g} \right) \alpha \beta}{(n_0^2 g^2 + n_0^2 g^2 L_0^2 m^2) \alpha^2 + n_1^2 m^2 \beta^2 - 2 n_0 n_1 g L_0 m^2 \alpha \beta}, \quad (10)$$

where $\alpha = \sin(gL)$ and $\beta = \cos(gL)$.

From the components of the matrix M , the expression of beam waist radius $\omega(z_w)$ of the focusing spot can be derived using Eq. (7). In addition, the Rayleigh range, Z_0 , can be expressed from the beam waist radius $\omega(z_w)$ as shown below,

$$Z_0 = \frac{n_2 \pi \omega^2(z_w)}{\lambda}. \quad (11)$$

Therefore, the mathematical expressions of characteristic parameters of a Gaussian beam focusing through the GRIN fiber probe, i.e., waist location z_w , waist radius $\omega(z_w)$, and Rayleigh range Z_0 , have been obtained as described by Eqs. (7), (10), and (11), respectively.

4 Impact of the Length of NCF and the Length of GRIN Fiber Lens on the Characteristic Parameters

From the obtained expressions of characteristic parameters, it can be found that the waist location, the waist radius, and the Rayleigh range depend on the length of the NCF, L_0 , and the length of the GRIN fiber lens, L , which is a periodical sine function about the product of g and L . In order to analyze the characteristic parameters more intuitively, MATLAB software is adopted to further solve these expressions in the following. In order to compare to the experimental data published in Refs. 14 and 15, relative design parameters are assigned below. The gradient constant of a GRIN fiber lens is set as $g = 5.5 \text{ mm}^{-1}$. The refractive indices of the SMF core, the NCF, and the center of a GRIN fiber lens are all 1.486. The index of air is $n_2 = 1$. The wavelength of the incoming Gaussian beam is $\lambda = 1300 \text{ nm}$. The beam waist radius at the input plane is $\omega_0 = 4.5 \text{ }\mu\text{m}$. In addition, some OCT or probe parameters are defined to tie to the beam parameters. The working distance is defined as the length from the output plane to the focal plane, i.e., focal distance from the lens surface. The spot size is defined as the full width at half maximum of the beam waist, i.e., beam waist diameter. The depth of field is defined as two times the Rayleigh range.

First of all, we analyze how the characteristic parameters change with the length of NCF L_0 given a constant length of the GRIN fiber lens $L_1 = 0.1 \text{ mm}$. Figures 3(a)–3(c) show the relationships between working distance, spot size, depth of field, and the length of NCF, respectively.

According to Fig. 3(a), we can obtain the following conclusions. 1. The working distance is a negative value when the NCF length is less than 0.28 mm. This phenomenon may be explained as below. On the one hand, when the NCF length is too short ($< 0.28 \text{ mm}$), it is not enough to expand the incoming beam due to the limited mode field diameter of the SMF ($9 \text{ }\mu\text{m}$). As a result, the beam always diverges through the whole NCF and GRIN fiber lens, which is similar to a “point source” entering the lens. On the other hand, the working distance is the calculated length from the output surface of the lens to the focal plane. Therefore, the point source appears to be a virtual focusing spot along the opposite direction of the beam propagating, which results in a negative value of the working distance as shown in Fig. 3(a). 2. For the NCF length of about 0.28 mm, the working distance is zero, which means the light propagating out from the GRIN fiber lens would be approximately collimated and the virtual focus spot would be located at the output surface of the lens. Then,

(a) Working distance

(b) Spot size

(c) Depth of field

Fig. 3 Impact of the length of the NCF on the characteristic parameters.

when the NCF length is greater than 0.28 mm, the light beam would begin to focus behind the output surface of the lens and the working distance quickly increase to 0.76 mm when the NCF length is increased up to about 0.37 mm. 3. If the NCF length is greater than 0.37 mm, the working distance decreases with the increase of the NCF length. This may be explained as in Ref. 9. The trajectory of propagating light within the GRIN fiber lens has three phases: divergence, access to the location having the maximum beam diameter, and focus. The larger incident beam diameter causes the beam

Fig. 4 Impact of the length of a GRIN fiber lens on the characteristic parameters.

to approach the maximum beam diameter within the GRIN fiber lens at a shorter distance, while the distance to reach the output surface of the GRIN fiber lens is extended. Finally, the light from the GRIN fiber lens is focused, producing a shorter working distance and a smaller beam diameter at the focal point.

Figures 3(b) and 3(c) demonstrate the impact of the NCF length on spot size and depth of field, respectively. Whether the value of working distance is positive or negative, both spot size and depth of field increase as the NCF length is added from 0 to 0.3 mm due to its effect of an expanding

Fig. 5 Schematic block diagram of properties testing system for GRIN fiber probes.

beam. When the NCF length is changed from 0 to 0.3 mm, the spot size increases from 10 μm to about 43 μm and the depth of field increases from 0.2 mm to about 2.2 mm. If the NCF length is greater than 0.3 mm, both spot size and depth of field decrease with the increase of NCF length resulting from the focusing of the lens.

As a special and useful case of interest, when the NCF length ranges from 0.32 to 0.4 mm, the working distance is located into 0.5 to 0.76 mm, and correspondingly the spot size into 26 to 40 μm , and the depth of field into 1.0 to 2.1 mm. Optimal designs of a GRIN fiber probe can be analyzed as follows. Generally speaking, one tends to design an OCT system with working distance and depth of field as long as possible and lateral resolution as high as possible, respectively. On the one hand, a probe has a limited detecting depth for OCT imaging when its working distance is less than 0.4 mm. On the other hand, spot size determines the lateral resolution of an OCT system to a great extent. The lateral resolution decreases with the increase of spot size. Therefore, in order to meet such different requirements of the OCT imaging probe, the use of NCF length, 0.32 to 0.4 mm, may be a good tradeoff, which means that the working distance is greater than 0.5 mm and the spot size less than 40 μm .

In the following, we further analyze how the characteristic parameters change with different lengths of the GRIN fiber lens given a constant NCF length $L_0 = 0.36$ mm. Figures 4(a)–4(c) show the relationships between working distance, spot size, depth of field, and the length of a GRIN fiber lens L_1 , respectively. According to Fig. 4, the following conclusions can be drawn.

First, the working distance shows a periodicity with an increase of the length of the GRIN fiber lens [see Fig. 4(a)], which can be explained through the term of pitch presented in Sec. 1. The gradient constant of the GRIN fiber is $g = 5.5 \text{ mm}^{-1}$, so the pitch P_L is about 1.14 mm according to formula (2), and 1/2-pitch about 0.57 mm, 1/4-pitch about 0.285 mm. It is not necessary to consider the direction of focus imaging since the light source is an axisymmetric Gaussian beam propagating through the single mode fiber into the fiber spacer (i.e., NCF) and then into the GRIN fiber lens, which means that the length period of the GRIN fiber lens for focusing the Gaussian beam is 1/2-pitch length. As a result, the working distance for the length ranges of a GRIN fiber lens from 0.57 mm (or 1/2-pitch) to 1.14 mm (1-pitch) is the same as the range from 0 to 0.57 mm.

Second, at the length of 1/4-pitch (0.285 mm) or multiple 1/4-pitch (2/4-pitch 0.57 mm, 3/4-pitch 0.855 mm, 1-pitch 1.17 mm, etc.), the GRIN fiber lens has a strong ability of focusing, which determines that the working distance is almost zero.

Third, the focal length may sharply increase to 0.76 mm within a very narrow range of the length of GRIN fiber lens about 0.1 mm, and then gradually decrease to 0 at the length of 1/4-pitch (0.285 mm). It is believed that this phenomenon results from the effect of the expanding beam of the NCF,

Table 1 Properties comparison between the testing values and the simulating data.

Types	Samples	Length of NCF (mm)	Length of GRIN fiber lens	Working Distance (mm)	Spot size (μm)	Depth of field (mm)
Experimental results (Refs. 14 and 15)	1	0	0.31	0.08	22	0.63
	2	0	0.33	0.16	19	0.45
	3	0	0.41	0.11	13	0.17
	4	0.36	0.10	0.65	31	1.1
	5	0.36	0.11	0.60	28	0.9
	6	0.36	0.12	0.50	23	0.60
Calculation results	1	0	0.31	0.08	21.5	0.56
	2	0	0.33	0.12	19.7	0.47
	3	0	0.41	0.10	12.8	0.20
	4	0.36	0.10	0.75	32.4	1.27
	5	0.36	0.11	0.63	23.7	0.68
	6	0.36	0.12	0.51	18.4	0.41

as well as the effect of the focusing beam of a GRIN fiber lens. When the length of the GRIN fiber lens locates into a small range of about 0.1 mm, the GRIN fiber lens begins to focus the incoming light beam and makes the focusing spot far away from the output plane due to the effect of expanding beam of the NCF. Then, focusing becomes deeper as the length of the GRIN fiber lens increases, which results in a shorter working distance down to 0 at 1/4-pitch length (0.285 mm).

Fourth, the value of the working distance appears negative when the length of the GRIN fiber lens locates into the range from 1/4- to 1/2-pitch length. And the plot of the working distance seems a centrosymmetric distribution about the plot from 0 to 1/4-pitch length. We believe that the incoming Gaussian beam will strongly focus (at the working distance of 0) when it goes through a path of 1/4-pitch length. Then, when the beam propagates through the GRIN fiber lens from the location of 1/4-pitch length to 1/2-pitch length, the beam begins to diverge from the focusing spot at the location of 1/4-pitch length. It is equivalent to a Gaussian beam, which enters a GRIN fiber lens from its output plane and is focused into a virtual spot at the location of 1/4-pitch length. However, the actual output plane of the GRIN fiber lens is regarded as the reference plane during the process of calculating the working distance, so the working distance shows negative values when the length of GRIN fiber lens ranges from 1/4-pitch length to 1/2-pitch length. This phenomenon may be termed as a conjugate imaging of the GRIN fiber lens. The problem of a negative working distance can be explained by Fig. 3(a) when the length of the GRIN fiber lens is less than 0.1 mm.

Finally, both the plot of the spot size and the plot of depth of field as shown in Figs. 4(b) and 4(c), respectively, also demonstrate the periodicity of a Gaussian beam focusing through the GRIN fiber lens. The analysis method is the

same as above. From Fig. 4, it can be concluded that, in the case of a certain length range of the GRIN fiber lens, both spot size and depth of field will be larger when the working distance increases, which means that the choice of the length of a GRIN fiber lens should be a tradeoff as the use of NCF.

5 Comparison Between Calculated Results and Experimental Data

Details about the fabrication and properties testing of GRIN fiber probes can be found in Refs. 14 and 15. Below is a brief overview of the related experimental design. A standard Corning SMF-28 single mode fiber is used as the principal light guide. An NCF from Prime Optical Fiber Corporation, Taiwan, is adopted as the fiber spacer, which is made of pure silica without a core but with the same cladding diameter as the SMF. A GRIN fiber, provided by Optical Fiber Solution, New Jersey, is employed as the focusing lens. The NCF was fusion spliced via arc welds to the Corning SMF-28 and then accurately cleaved to a theoretical length. The GRIN fiber was then fusion spliced to the cleaved NCF and precisely cleaved at a precalculated length.

In order to characterize the beam propagating through GRIN fiber probes, Fig. 5 shows a schematic diagram of the properties detection system designed in Refs. 14 and 15, which consists of a beam profile measurement system (BeamView Analyzer, Oregon) with an infrared CCD camera (Electrophysics, New Jersey) and a superluminous diode source (Covega, Maryland) with 60 nm 3 dB bandwidth at 1300 nm center wavelength. A 40 JIS (Japanese industrial standard) microscopic objective lens and a related objective tube were attached to the input window of the camera to increase the image resolution. During the measurement process of the working distance and spot size, the distribution of light intensity at various distances along the direction of

Table 2 Focusing properties comparison between different designs of GRIN fiber probes.

Groups	NCF length (mm)	Length of GRIN			Depth of field (mm)
		fiber lens (mm)	Working distance (mm)	Spot size (μm)	
1	0	0.31	0.08	21.5	0.56
		0.33	0.12	19.7	0.47
		0.41	0.102	12.8	0.20
2	0	0.88	0.075	21.6	0.56
		0.90	0.12	19.8	0.47
		0.98	0.102	12.9	0.20
3	0	1.45	0.072	21.7	0.57
		1.47	0.11	19.9	0.48
		1.55	0.103	12.9	0.20
4	0.36	0.10	0.75	32.43	1.27
		0.11	0.63	23.72	0.68
		0.12	0.51	18.37	0.41
5	0.36	0.67	0.76	33.79	1.38
		0.68	0.65	24.55	0.73
		0.69	0.52	18.89	0.43
6	0.36	1.24	0.77	35.22	1.50
		1.25	0.66	25.43	0.78
		1.26	0.54	19.44	0.46

propagation after the lens was first accurately measured by the beam profile system with the horizontal resolution of $1.0\ \mu\text{m}$ and the vertical resolution of $1.1\ \mu\text{m}$, respectively. Then, the working distance and $1/e^2$ spot size were obtained from the measured intensity distribution.

Table 1 shows the properties comparison between the testing data obtained from Refs. 14 and 15 and the calculation data from the expressions presented above. It is obvious that the calculation results of working distance, spot size, and depth of field are almost the same as the experimental data. Therefore, the presented analytical method for designing a GRIN fiber probe is feasible and effective based on the characteristics of beam propagating within the probe. In terms of the differences between the experimental and calculation results, there may be the following reasons. First, the cutting lengths of the NCF and GRIN fiber lens cannot be completely precisely cleaved at the precalculated lengths, which results in errors of beam expanding by NCF and focusing by the lens between the experimental and calculation data. Second, the actual difference between refractive index of the NCF and center of the GRIN fiber lens is not taken into account, which would lead to differences between the experimental results and the calculation data in this paper. Third, the measurement error of the experimental system is another reason. There may be other factors to be investigated.

The validation of the mathematical expressions of characteristics is verified above for the design of the GRIN fiber probe. As another important prediction conclusion, the pitch length of the GRIN fiber lens is discussed as follows for optimal design of GRIN fiber probes. Since the length period of a GRIN fiber lens is $1/2$ -pitch length for the focusing of a Gaussian beam, the characteristic parameters will be kept the same when the GRIN fiber lens increases every $1/2$ pitch length. Therefore, the requirement on the fabrication process of the ultrasmall optical probes could be reduced by increasing the length of the GRIN fiber lens properly. Table 2 shows some calculation data from the derived expressions in this paper. We can find that the working distance, spot size, and depth of field seem to be equal given the constant length of the NCF 0 or 0.36 mm and the periodically increasing length of the GRIN fiber lens. For example, setting the NCF length as 0.36 mm, the length of the GRIN fiber lens is chosen as 0.11, 0.68, and 1.25 mm, respectively; the working distance is 0.63, 0.65, and 0.66 mm, respectively; the spot size 23.72, 24.55, and 25.43 μm , respectively; and the depth of field is 0.68, 0.73, and 0.78 mm, respectively. The difference of focusing properties between these different probes is very little, which means the pitch length of the GRIN fiber lens can be used to optimize the design of GRIN fiber probes by increasing the lens length periodically.

6 Conclusions

The study of GRIN fiber probes is an important topic for the miniaturization of an OCT system. The design theory of a GRIN fiber probe has been largely overlooked, although there have been a few articles published about the fabrication or application of such probe. In this paper, an analytical method was presented for designing GRIN fiber probes. Based on the presented model of the GRIN fiber probe, the complex beam parameter matrix transformation method was adopted to solve the mathematical expressions of characteristic parameters of a Gaussian beam propagating through the probe. MATLAB software was used to analyze the impact of the length of NCF and the length of GRIN fiber lens on the characteristic parameters. Properties comparison was performed between the simulating results and the experimental data published previously. In addition, optimal designs of the GRIN fiber probe were discussed based on the derived expressions.

From this article, we give the following conclusions. First, the mathematical expressions of characteristic parameters of a Gaussian beam propagating through a GRIN fiber probe can help analyze and design such an ultrasmall probe with special demands of optical properties. Second, a GRIN fiber lens has a period of 1/2-pitch length for focusing a Gaussian beam, which may be helpful to decrease the fabrication requirements of GRIN fiber probes by increasing the length of the GRIN fiber lens properly. Third, the use of NCF can improve the focusing performance of a GRIN fiber probe by means of its effect of expanding beam. It is noted that the choice of the length of NCF and the length of the GRIN fiber lens should be a tradeoff considering many factors such as working distance, lateral resolution, and depth of field, etc.

Although the presented analytical method based on the mathematical expressions of characteristics was verified to be effective for designing GRIN fiber probes in this paper, there are a few important issues left to be researched. For example, the single-to-noise ratio of the OCT system related to the GRIN fiber probe should be analyzed since the sensitivity is a crucial aspect for OCT performance. Some helpful analysis methods and results can be found in Refs. 17–21. In addition, the relationship between the Gaussian beam characteristics (e.g., waist location, waist size, Rayleigh range) and the OCT parameters (e.g., working distance, lateral resolution, and depth of field) should be further investigated quantitatively. Last, but not the least, it is necessary to perform experimental studies on the fabrication and properties measurement of a GRIN fiber probe. For example, standard deviation of several measurements of working distance or spot size may need to be solved to study the problems about measurement error.

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