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**DETERMINATION OF MINIMUM LIMITING AXIAL DEPTH OF CUT FOR
2½D POCKET MACHINING BASED ON RECEPTANCE COUPLING
(Determination of limiting axial depth for 2½D pocket)**

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ABSTRACT

A minimum limiting axial depth of cut guarantees stable machining for any cutting speed and selected as height of the z -level of cut used for 2½D pocket machining. It is determined based on the frequency response function (FRF) and receptance coupling sub-structure analysis (RCSA) to minimize experimentation. For process planning considerations, long cutters are more preferable for deep pockets to reduce the number of z -levels of cut. However, the higher cutter flexibility results in bouncing effect during impact testing. To overcome this problem, a string test is proposed in which the cutter tip is deflected by a string and receptance measured for free vibration. In this work, the FRF at the tip of cutters with different geometries was measured with both the tests to study the effect of geometry on receptance. The prediction of experimentally calibrated RCSA models shows good agreement with the measured receptance. The results also suggest the suitability of string test for flexible cutters. Finally, a reference plot of the variation of limiting axial depth with cutting geometry and conditions is generated based on the predicted receptance. The plot provides *a priori* determination of the stable limiting axial depth of cut which facilitates the automatic generation of process plans for 2½D pocket machining.

Keywords: 2½D pocket machining; minimum limiting axial depth; frequency response function; receptance coupling; string test; cutter geometry.

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INTRODUCTION

In 2½D pocket machining, 2D corresponds to the movement of the cutter in the XY plane whereas the ½D accounts for cutter movement in the z-direction. Thus the entire pocket height (h) can be divided into a number (n_a) of axial depths or z-levels of cut as shown in Figure 1. To reduce the machining time, n_a should be low but at the same time it is dictated by a limiting condition for stable machining. Unstable machining occurs mainly due to regenerative chatter (Koenigsberger and Thusty, 1967; Tobias, 1965; King, 1985) caused by the variation in the chip thickness when the cutter removes material from an undulated surface generated in its previous pass. It deteriorates the machined surface quality and may also cause failure of one or more machine components. It is well known that chatter should be avoided for a safe and efficient machining. Thus the limiting condition is obtained to avoid the onset of chatter during cutting process. For end-milling there exists a limiting axial depth of cut, a_{lim} (Koenigsberger and Thusty, 1967; Tobias, 1965; King, 1985; Merrit, 1965) which depends on the dynamic characteristics of the machine tool mechanical structure as well as the cutter geometry, the material properties of cutter and workpiece combination and the cutting conditions. The selection of the axial depth of cut under a predetermined a_{lim} would help in increasing productivity of machining process by achieving stable machining. For this purpose, stability lobes (Koenigsberger and Thusty, 1967; Tobias, 1965; King, 1985; Merrit, 1965; Thusty *et al.*, 1983) are often used, e.g., Figure 1 shows the plot of a_{lim} for different cutting speeds for a particular machine tool which divides the machining process into stable and unstable regions.

[Insert Figure 1 about here]

In the context of 2½D pocket machining, the maximum axial depth of each z-level should be selected as a_{lim} to minimize n_a and subsequently the machining time. Selecting a longer cutter will be intuitive to machine deep pockets. But longer cutter is more flexible which causes the onset of chatter at lower a_{lim} (Tlusty *et al.*, 1996; Smith *et al.*, 1998) and also results in higher cutter deflection during machining. Hence, the determination of a_{lim} based on the known process inputs becomes an important step for developing process plans for the pocket machining. A plot of three-dimensional stability surfaces along tool geometry (Schmitz *et al.*, 2004) had been presented. In this work a similar plot will be generated to show the relationship between limiting a_{lim} tool geometry and cutting conditions.

Minimum limiting axial depth of cut

The determination of a_{lim} and generation of the corresponding stability lobes, require the knowledge of dynamics of mechanical structure of the machine tool at cutter tip as well as cutting process. When cutting speed is used within conventional range (less than 15,000 rpm), the stability lobes are observed to be quite close to each other. In this range, a minimum limiting axial depth of cut (a_{lim}^{min}) of a cut as shown in Figure 1 can be selected to quantify the stability limit of end milling process. The advantage of using the limit is that it is independent of the cutting speed and a simplified formulation (Koenigsberger and Tlusty, 1967) based on Nyquist stability criterion can be used to determine it, as follows,

$$a_{lim}^{min} = \frac{1}{2 \cdot K_s \cdot \frac{N}{2} \cdot \frac{d_r}{d} \cdot u \cdot \min(\text{Re}[G_{11}(j\omega)])} \quad (1)$$

where K_s : specific cutting coefficient; N : number of tooth; d_r : radial depth of cut; d : diameter of cutting cutter; u : directional orientation factor, which takes into account the orientated receptance through the angle subtended by the displacement and force vectors; $\min(\text{Re}[G_{11}(j\omega)])$: minimum real part of the direct receptance at the cutter tip.

The value of $\min(\text{Re}[G_{11}(j\omega)])$ is calculated based on the FRF measured at the cutter tip. The FRF can be experimentally measured by several methods (Ewins, 1995) including impact test, drop test, pendulum test, and exciters. In these methods a known exciting force is applied to the structure and the response is measured with help of sensors. The means of providing the excitation force differs in each method. The sensors used for measuring vibration response are generally accelerometers, velocity and displacement transducers.

This paper focuses on determination of the minimum real part of receptance and subsequently the estimation of minimum limiting axial depth of cut for different cutter geometries. For this purpose, a brief discussion of receptance and RCSA used to avoid repetitive experimentation is given. The FRF measurement at the cutter tip with impact test is discussed; the associated problem for flexible cutters is identified and a string test is proposed to avoid the above problem. The experimental set-up is described, and results and discussion of experimentation and prediction of receptance with RCSA are provided. The results include FRF measurements at SHC and cutter tip for different cutter geometries along with comparison of predicted and measured receptances at the cutter tip. Based on RCSA, a reference plot of the minimum limiting axial depth of cut for different cutter geometry and cutting condition is also presented. Finally, conclusions and future work are provided.

RECEPTANCE AND RECEPTANCE COUPLING

It is possible to analyze inherent dynamic properties of a mechanical structure from the response signal measured by vibrating it with a known excitation force. For this purpose the ratio of the frequency spectrum of response to frequency spectrum of the excitation signal within a specific frequency range of interest, called frequency response function (FRF) is determined. A particular form of FRF, where response in displacement, $X(j\omega)$, and excitation force, $F(j\omega)$, is called receptance, $G(j\omega)$. The concept of receptance finds tremendous application in determining dynamic characteristics of the machine tool mechanical structure and $a_{\text{lim}}^{\text{min}}$ (Equation (1)). For simplicity, the cutter and machine tool mechanical structure assembly is considered as a single degree of freedom system such that the system can vibrate only in one direction (say along direction 1). Then the direct receptance $G_{11}(j\omega)$ obtained by applying force and measuring displacement both in direction 1 can be expressed as (Koenigsberger and Tlustý, 1967),

$$G_{11}(j\omega) = \frac{X(j\omega)}{F(j\omega)} = \frac{\omega_n^2}{k \cdot (\omega_n^2 - \omega^2 + 2j\zeta\omega_n\omega)} \quad (2)$$

where m (mass), k (stiffness) and c (damping coefficient) are the modal parameters, ω_n is natural frequency given by $\omega_n = \sqrt{k/m}$; ζ is damping ratio given by $\zeta = c/2\sqrt{km}$.

The cutter geometry has significant effect on FRF and receptance at the cutter tip. This would necessitate repetition of FRF measurements at the cutter tip, for different cutters inserted in the SHC. To avoid these repetitive measurements, researchers have used receptance coupling sub-structure analysis or RCSA (Schmitz and Duncan, 2005; Schmitz *et al.*, 2004; Schmitz *et al.*,

2001; Schmitz and Donaldson, 2000; Park *et al.*, 2003; Kivanc and Budak, 2004) to predict the dynamics of SHC and cutter assembly. In RCSA, the machine tool structure is first divided into two substructures, SHC and cutter, shown in Figure 2a (Schmitz *et al.* 2001). These substructures are assumed to be connected by linear and torsional spring and damper system to imitate the actual connection to give the SHC-cutter assembly as shown in Figure 2b. As it is very difficult to experimentally measure receptance of the free-free cutter, researchers have resorted to analytical (Schmitz and Duncan, 2005; Schmitz *et al.*, 2004; Schmitz *et al.*, 2001; Schmitz and Donaldson, 2000; Bishop and Johnson, 1960) as well as finite element methods (Park *et al.*, 2003; Kivanc and Budak, 2004). Once the receptances of individual substructures are known, the unknown connection coefficients (k_x , k_θ , c_x and c_θ), are determined by comparing measured and RCSA based predicted receptances at the cutter tip.

[Insert Figure 2 about here]

The relevant formulations for RCSA are provided in Table 1, where the receptances of individual components are denoted by H and that of the SHC-cutter assembly as G . The direct ($m=n$) and cross ($m \neq n$) receptance terms of displacement under applied force (H_{11} , H_{22} and $H_{12}=-H_{21}$), displacement under applied moment (L_{mn}), rotation under applied force (N_{mn}) and the rotation under applied moment (P_{mn}) are derived analytically (Bishop and Johnson, 1960). For the purpose of calculation, density (ρ), elastic modulus (E), viscous damping co-efficient (c) for the HSS cutter material and the second moment of inertia (I) are required. The dimensionless frequency parameter (λ_r) for r^{th} mode, can be derived for the free-free beam modes with following values: Mode 1: 4.73004, Mode 2: 7.83520, Mode 3: 10.9956, Mode 4: 14.1372, Mode 5: 17.2788 and Mode $r>5$: $(2r+1)\pi/2$ (Blevins, 1979). The viscous damping introduces velocity

terms in the analytical formulation of the cutter's receptance. For this reason, linear and torsional ratio of frequency spectrum of the velocity and applied force, called mobility (H'_{mn} , L'_{mn} , N'_{mn} and P'_{mn}) are included (Ewins, 1995).

[Insert Table 1 about here]

The connection parameters of linear stiffness and damping coefficient (k_x and c_x) as well as torsional stiffness and damping coefficient (k_θ and c_θ) are estimated by comparing the experimentally determined G_{11} and the predicted $\hat{G}_{11}$. The two receptances within frequency range of interest are fitted with a non-linear least square method. The connection parameters are estimated for extreme cutter geometries and used for prediction of receptances for the intermediate geometries. The result and discussion of relevant experimentation and prediction of receptances are presented later in the paper.

MEASURING THE FREQUENCY RESPONSE FUNCTION

The impact test is widely used for measuring the FRF of a machine tool structure due to its portability, accurate force measurements and wide excitation frequency range. A known impact force, $F(t)$ is applied on the mechanical structure and the resulting response, $X(t)$ can be measured with an accelerometer, laser transducer, etc. The Fourier Transform of force ($F_k \equiv F(nT)$) as a discretized signal and response ($X_k \equiv X(nT)$) as a discretized signal can be represented as (Ewins, 1995),

$$\left. \begin{aligned} F_n(j\omega) &= \frac{1}{N} \sum_{k=1}^N F_k \cdot e^{-j \cdot 2\pi nk/N} \\ X_n(j\omega) &= \frac{1}{N} \sum_{k=1}^N X_k \cdot e^{-j \cdot 2\pi nk/N} \end{aligned} \right\} \quad (3)$$

where T is the time interval, N is the total number of samples and n is an index varying from 1 to N . The signal processing for the FRF measurement with impact test includes calculation of auto-spectrum of force signal ($S_{FF}(j\omega)$), cross-spectrum of displacement and force signal ($S_{XF}(j\omega)$); and auto-spectrum of noise in force signal ($S_{NN}(j\omega)$). The transfer function of receptance is given below,

$$G(j\omega) = \frac{S_{XF}(j\omega)}{S_{FF}(j\omega) + S_{NN}(j\omega)} \quad (4)$$

where, $S_{XF}(j\omega) = X(j\omega) \times F^*(j\omega)$; $F^*(j\omega) = \text{conjugate}(F(j\omega))$; $S_{FF}(j\omega) = F(j\omega) \times F^*(j\omega)$;

$S_{NN}(j\omega) = N(j\omega) \times N^*(j\omega)$; $N^*(j\omega) = \text{conjugate}(N(j\omega))$;

The force profile generated with impact hammer during impact test can be divided into three phases,

- I) The hammer traveling into the cutter and deflecting it, shown in Figure 3(I)
- II) The cutter at its extreme deflected position, with the maximum deflection represented as $\delta_{\max}$, shown in Figure 3(II)
- III) The hammer traveling out of cutter with the later starting free vibration, shown in Figure 3(III)

[Insert Figure 3 about here]

The response of the cutter in terms of displacement is transient in nature until time t_1 when the hammer is in contact with cutter. In this duration, the response corresponds to the combination of dynamics of the hammer as well as its non-linear contact with the cutter. Beyond time t_1 , the cutter vibrates freely, which corresponds entirely to the dynamics of cutter due to the impulse loading. Hence it is important to consider the response signal beyond the time t_1 to measure the FRF of the cutter.

Issues of Impact test

During impact testing carried out for a long slender cutter with higher flexibility, the cutter deflects at a faster rate than the hammer in phase I as shown in Figure 3(I). This causes the cutter to reach phase II quicker than the hammer. As the cutter recovers from its maximum deflected position in phase II, it hits back the hammer still travelling into the cutter, which leads to multiple hits. The force signal consists of multiple impulse peaks which is referred to as a bouncing effect or multiple hits (Ewins, 1995) as is shown in Figure 4. The multiple impulse peaks of shorter time duration are observed rather than a single peak of longer duration which reduces the excitation bandwidth. The time gaps between the initial impact and the rebounds result in an erroneous FRF measurement, which becomes the ratio of very low levels of response and excitation force at certain frequencies. This is commonly referred as holes (Ewins, 1995) and introduces errors in the subsequent calculation of receptance. Although the bouncing effect can be reduced by using a softer tip like nylon tip instead of harder steel tip on the hammer but it cannot be completely avoided for slender tools. The displacement response of multiple hits also includes noise resulting from dynamics of the hammer and non-linearities of multiple cutter-

hammer contacts.

[Insert Figure 4 about here]

String test

During the measurement of FRF of flexible cutters which are required to machine deep pockets, it is beneficial to avoid the bouncing effect observed in impact tests. Moreover, when hammer excites the cutter at a higher frequency, the secondary modes of vibration may become large and lead to erroneous $\min(\text{Re}[G_{11}(j\omega)])$. The resulting errors in FRF measurements can be reduced by replacing dynamic force excitation with a static one. For this purpose, a string test is proposed where a known static force is applied on the flexible cutter with the help of a string. The string is cut and the corresponding displacement response is measured to obtain the FRF. The string test is similar to ‘Step Relaxation’ type of transient excitation (Ewins, 1995) commonly used where impact loading is not suitable.

The static force applied to the cutter (F_{static}) has a step profile, as shown in Figure 5. It can be measured with help of a force sensor for example spring scales, load cells, piezo-electric transducers, etc. The force profile of the string test after the string is cut, entirely corresponds to phase III of the impact test shown in Figure 3(III). This completely avoids the bouncing effect and non-linearity of phases I and II of the impact test. In this work, the applied static force, F_{static} used to pull the cutter by a string is measured with a spring scale, which generates the static displacement of δ_{static} . When the string is cut, the cutter vibrates freely and the vibration response entirely represents the cutter dynamics. Fourier spectrum of the vibration response given as $X(j\omega)$ is divided by the magnitude of F_{static} to obtain the FRF of cutter ($G_{\text{ST}}(j\omega)$), given below.

$$G_{ST}(j\omega) = \frac{X(j\omega)}{F_{static}} \quad (5)$$

The frequency content of $G_{ST}(j\omega)$ corresponds to that of $X(j\omega)$. The value of $G_{ST}(j\omega)$ at $\omega = 0$ corresponds to the static receptance, defined as, $G_{static} = \delta_{static}/F_{static} = G_{ST}(j\omega)|_{\omega=0}$ which indicates the value of flexibility of cutter under static loading. For stiffer structures like SHC, it may not be possible to provide enough energy with the string to excite the structure. For such cases impact test will be more suitable. Care must be taken to align the force applied through string. The string should be perpendicular to cutter axis for which a rigid frame with pulleys can be attached to the machine tool structure.

[Insert Figure 5 about here]

EXPERIMENTAL SETUP AND PROCEDURE

In this work, a Barer Proteo 5-axis horizontal machining center is used with HSS flat-end milling cutters of different geometries. An impact hammer (model: 086C03, PCB Piezotronics) was used along with a charge amplifier (model: 482A17, PCB Piezotronics) for the impact test. The displacement response of the vibration was measured with a laser sensor (model: LTS 15/2.9, LMI Inc.) giving both good static and high frequency responses. Displacement sensor is preferred to the conventional accelerometer, as the acceleration signal has to be integrated twice to obtain the required displacement. This introduces integration constants resulting in undesirable drift in displacement signal. Both the force and vibration responses are measured and stored through an oscilloscope (model: HP 35670A, Agilent Inc.). The sampling frequency of 65.54 kHz is used and the corresponding Nyquist frequency being 32.77 kHz. The experiments are

repeated and averaged to eliminate measuring errors. The maximum amplitude of force for both impact and string tests are maintained within $\pm 5\%$ deviation. The block diagram of the experimental set-up is shown in Figure 6a. The FFT of the force input and displacement response signal and $\min(\text{Re}[G_{11}(j\omega)])$ is obtained with help of MATLAB. The FRF is measured at the SHC (position 3) with help of impact test to obtain $\min(\text{Re}[H_{33}(j\omega)])$, while both impact and string tests are used to measure $\min(\text{Re}[G_{11}(j\omega)])$ at the cutter tip (position 1) for different cutter length and diameter.

Also, in order to confirm applicability and acceptability of string test for dynamic analysis of machine tool mechanical structure and its components, free vibration responses of both impact and string tests were compared in time and frequency domains confirming high (more than 80%) correlation between them. As an example, Figure 6b shows typical free vibration responses at cutter tip for impact and string tests having a coefficient of correlation of 0.81.

[Insert Figure 6 about here]

Separate hexagonal attachments are fabricated for obtaining a plane surface on the cylindrical SHC as well as on the cutter tip. The attachments guarantee mutual orthogonality of the measurement directions XX and YY, as shown in Figure 7a and 7b respectively. The attachments are made of aluminium and designed such that they do not interfere with the dynamic characteristics of the SHC and cutter. A maximum variation of 5% has been observed in the natural frequency of cutter with and without tip mass, for the lowest modal mass cutter. For position 3, the forces F_{3-XX} and F_{3-YY} as well as the direct displacement responses x_{3-XX} and

x_{3-YY} are mutually orthogonal. The triggering point of the measurement decides the starting point of simultaneous capture of force and displacement response signals. It is set on the force and acceleration signal for the impact and string test, respectively.

[Insert Figure 7 about here]

RESULTS AND DISCUSSION

FRF measurements at spindle-holder combination and cutter tip

The FRF at the SHC and cutter tip are measured with impact test. The string test is used only to measure FRF at the cutter tip. The magnitude of the FRF measured at the SHC and HSS end milling cutter tip with diameter (d) of 12.70 mm and length (L) of 42 mm is shown in Figure 8a. As the cutter is more flexible than the SHC, $\min(\text{Re}[G_{11}(j\omega)])$ is much higher than $\min(\text{Re}[H_{33}(j\omega)])$ shown in Figure 8b. It can also be observed in Figure 8a that the primary mode of vibration of the SHC, denoted by mode A has a natural frequency of $\omega_n^1=1890$ Hz. When the cutter is inserted in the SHC, the primary mode changes to mode B with $\omega_n^1=1334$ Hz and 1297 Hz for impact and string test respectively. This suggests the addition of a vibration mode of the machine tool structure when the cutter is inserted into the SHC. Mode B will dominate as the cutter is more flexible than the SHC and the difference between the two modes will be higher for more flexible cutters. FRF measured at the cutter tip with impact and string tests are found to be similar with the string test giving a higher value of $\min(\text{Re}[G_{11}(j\omega)])$. It can be caused by the lower static force compared to the impact force which reiterates that the string test may not provide enough energy to excite stiffer structures.

[Insert Figure 8 about here]

The value of $\min(\text{Re}[G_{11}(j\omega)])$ and natural frequencies (ω_n^I) calculated based on FRF measurements at the tip of cutter with different geometries using both impact and string test are tabulated in Table 2. It can be observed in the table that receptances measured with both tests are similar. It is believed that deviations arise from difference in the magnitude and nature of the exciting force signals. The measurement of direct receptance obtained with string test will be more reliable for cutters with high flexibility as the bouncing effect is completely avoided. But for stiffer cutters, impact tests will give reliable result.

The Table 2 also shows the effect of cutter geometry on $\min(\text{Re}[G_{11}(j\omega)])$ for five different cutter length and diameter. It is observed that for a particular cutter diameter ($d=12.7$ mm), as the length decreases (Case 1 to 3), $\min(\text{Re}[G_{11}(j\omega)])$ decreases while ω_n^I increases. This occurs because flexibility of the cutter increases with the decrease in the length. Again, with the decrease in the diameter (case 3 to 5) at a particular length ($L=42$ mm), $\min(\text{Re}[G_{11}(j\omega)])$ increases while ω_n^I decreases as flexibility of the cutter increases. Thus the variation of the measured FRF for different cutter geometries shows a logical trend and is used to estimate the coupling coefficients of RCSA as described in the next section.

[Insert Table 2 about here]

Receptance coupling substructure analysis

Receptance coupling substructure analysis (RCSA) is implemented for different cutter geometries. The predicted and measured receptances are obtained at tip of two typical cutter

geometries (case 1 and 3) with both impact and string tests. The four connection parameters, k_x , k_θ , c_x and c_θ are estimated by fitting the measured and predicted receptances within frequency range of interest. This is done using a non-linear least square method with MATLAB and the result is presented in Table 3. It can be observed in the table, that values of stiffness coefficients (k_x and k_θ) obtained with impact and string tests are closer than the torsional damping (c_x and c_θ). This suggests that moment applied during string test results in very low linear (x) and angular (θ) responses which subsequently generates lower mobilities (G'). As direct receptance (G_{mm} or G_{nn}) influences the calculation of a_{lim}^{min} , lower mobilities are acceptable for this work. But for applications where cross-receptances (G_{mn} or G_{nm}) or mobilities play an important role, the string test may not provide accurate result.

[Insert Table 3 about here]

The comparisons of the predicted and measured receptance of the cases 1 and 3 with impact and string test are shown in Figures 9 and 10 respectively. The cutter considered in case 1 has higher length to diameter ratio and hence higher flexibility than the cutter considered in case 3. In the figures, it is observed that the measured and predicted receptances for each test are close to each other. The deviations can be attributed to the difference in clamping torques applied to the cutters, simplistic signal processing, approximation of cutter geometry, deviation in contact conditions and finite repeatability of FRF measurements (Schmitz *et al.*, 2001). It can also be observed that string test measures the primary mode of vibration of the cutter better than impact test for the flexible cutter (case 1) in Figure 10a. Whereas the impact test measurement is closer to the predicted receptance for stiffer cutter (case 3) in Figure 9b. This supports the use of string test to avoid bouncing effect observed while performing impact test on flexible cutters. The

overall agreement of predicted and measured receptances obtained from both impact and string tests are found to be adequate. Hence receptances of intermediate cutter geometries ($95\text{ mm} \geq L \geq 42\text{ mm}$ and $12.7\text{ mm} \geq d \geq 6.35\text{ mm}$) can be satisfactorily predicted using the estimated coupling coefficients of RCSA.

[Insert Figure 9 about here]

[Insert Figure 10 about here]

Reference plot for limiting axial depth

The value of $a_{\text{lim}}^{\text{min}}$ can be determined based on Equation (1), with the $\min(\text{Re}[G_{11}(j\omega)])$ calculated from FRF measurements and/or predicted with RCSA for different cutter geometries. The $a_{\text{lim}}^{\text{min}}$ for intermediate cutter geometries lying within the range considered for measurement ($95\text{ mm} \geq L \geq 42\text{ mm}$ and $12.7\text{ mm} \geq d \geq 6.35\text{ mm}$) are calculated based on RCSA. Mild steel workpiece and a HSS cutter with $N=4$; $d_r=d$ (slot cut) and $d_r=d/4$ (peripheral cut), $K_s=3200\text{ N/mm}^2$ and $\nu=0.342$ (King, 1985) are used for calculation. The resulting three-dimensional reference plot of $a_{\text{lim}}^{\text{min}}$ along with cutter length and diameter is shown in Figure 11. It can be observed that as stiffness of the cutter increases, $a_{\text{lim}}^{\text{min}}$ also increases. This happens with decrease in length and/or increase in diameter. The plot will play an important role in providing prior knowledge of chatter-free limiting depth thus facilitating subsequent process planning. Although the value of $a_{\text{lim}}^{\text{min}}$ is quite low for slot cut, but for the peripheral cut it is comparable to practical recommendations. To extend the applicability, plots of $a_{\text{lim}}^{\text{min}}$ for several other radial depths can be included.

[Insert Figure 11 about here]

CONCLUSIONS

In this work, the problem of selecting the height of each z -levels of cut for 2½D pocket machining is addressed. For this purpose, a minimum limiting axial depth of cut which guarantees stable machining for any cutting speed is determined based on receptance coupling using RCSA. A simplified formulation is implemented which requires the minimum real part of the FRF to be measured or predicted at the cutter tip. The issue of bouncing effect during the measurement of FRF for flexible cutters with impact tests is addressed. To avoid it, a static string test is proposed and successfully implemented.

The FRF at the cutter tip is measured with both impact and string test, while FRF at the SHC is measure with impact test. The FRF measurement for different cutter geometry shows that when the minimum real part of the direct receptance increases with flexibility of the cutter and vice versa. A shift in the frequency of the primary mode of vibration is also observed for different cutter geometries and SHC. The FRF measurements at the cutter tip are also used to estimate the coupling coefficients of RCSA. The predicted receptances for both impact and string tests are found to be close with the measurements. Moreover for the flexible cutter the prediction is closer to the string test measurement whereas for the stiffer cutter the prediction based on the impact test measurement is closer. Hence it can be concluded that the string test helps in avoiding the bouncing effect for flexible cutters, but for stiffer cutters it may not provide enough excitation energy. Based on RCSA, a reference plot of the variation of the minimum limiting axial depth of cut with different cutter geometry and cutting conditions is generated. The plot facilitates the automatic generation of process plans for 2½D pocket machining by providing a

priori determination of the chatter free axial depth of cut that will lead to stable machining.

The difference between the measured and predicted receptances can be reduced by maintaining a consistent clamping torque with a torque-wrench, using more sophisticated signal processing and accurate knowledge of cutter geometry and automating the loading scheme to improve the repeatability of the FRF measurements. Reference plots of minimum limiting axial depth of cut for different cutting conditions and cutter and workpiece materials can also be generated in the similar way.

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TABLE 1 Formulation of Receptance Coupling Substructure Analysis

Functions	Formulations
<i>Receptances in series form</i> (Bishop, 1960)	$H_{mn}(j\omega) = \frac{x_m(j\omega)}{f_n(j\omega)} = \sum_{r=0}^{\infty} \frac{\phi_r(x_m) \phi_r(x_n)}{-\rho L \omega^2 + jcL\omega + \frac{EI\lambda_r^4}{L^3}}; \quad L_{mn}(j\omega) = \frac{x_m(j\omega)}{m_n(j\omega)} = \sum_{r=0}^{\infty} \frac{\phi_r(x_m) \phi_r'(x_n)}{-\rho L \omega^2 + jcL\omega + \frac{EI\lambda_r^4}{L^3}};$ $N_{mn}(j\omega) = \frac{\theta_m(j\omega)}{f_n(j\omega)} = \sum_{r=0}^{\infty} \frac{\phi_r'(x_m) \phi_r(x_n)}{-\rho L \omega^2 + jcL\omega + \frac{EI\lambda_r^4}{L^3}}; \quad P_{mn}(j\omega) = \frac{\theta_m(j\omega)}{m_n(j\omega)} = \sum_{r=0}^{\infty} \frac{\phi_r'(x_m) \phi_r'(x_n)}{-\rho L \omega^2 + jcL\omega + \frac{EI\lambda_r^4}{L^3}};$
<i>Direct Receptances</i> ($m=n$) (Bishop, 1960)	$H_{11} = \frac{-4}{\rho AL \omega^2} + \sum_{r=1}^{\infty} \frac{\phi_r(L) \phi_r(L)}{-\rho L \omega^2 + jcL\omega + \frac{EI\lambda_r^4}{L^3}}; \quad L_{11} = \frac{-6}{\rho AL^2 \omega^2} + \sum_{r=1}^{\infty} \frac{\phi_r(L) \phi_r'(L)}{-\rho L \omega^2 + jcL\omega + \frac{EI\lambda_r^4}{L^3}};$ $N_{11} = \frac{-6}{\rho AL^2 \omega^2} + \sum_{r=1}^{\infty} \frac{\phi_r'(L) \phi_r(L)}{-\rho L \omega^2 + jcL\omega + \frac{EI\lambda_r^4}{L^3}}; \quad P_{11} = \frac{-6}{\rho AL^2 \omega^2} + \sum_{r=1}^{\infty} \frac{\phi_r'(L) \phi_r'(L)}{-\rho L \omega^2 + jcL\omega + \frac{EI\lambda_r^4}{L^3}};$
<i>Characteristic function for free-free beam</i> (Bishop, 1960)	$\phi_r(x) = \cosh\left(\frac{\lambda_r x}{L}\right) + \cos\left(\frac{\lambda_r x}{L}\right) - \left(\frac{\cosh(\lambda_r) - \cos(\lambda_r)}{\sinh(\lambda_r) - \sin(\lambda_r)}\right) \times \left(\sinh\left(\frac{\lambda_r x}{L}\right) + \sin\left(\frac{\lambda_r x}{L}\right)\right)$
<i>Direct Mobilities</i> (Schmitz <i>et al.</i> , 2001)	$H'_{11} = j\omega \cdot H_{11}; \quad N'_{11} = j\omega \cdot N_{11}; \quad L'_{11} = j\omega \cdot L_{11}; \quad P'_{11} = j\omega \cdot P_{11};$
<i>Receptance of the Assembly</i> (Schmitz and Donaldson, 2000)	$\hat{G}_{11} = \frac{X_1}{F_1} = H_{11} - H_{12} C_1^{-1} C_2 - L_{12} C_3^{-1} ((k_{\theta} N_{21} + c_{\theta} N'_{21})) - C_4 C_1^{-1} C_2$ <p>where</p> $C_1 = (k_x H_{33} + k_x H_{22} + c_x H'_{33} + c_x H'_{22} + 1) - C_3^{-1} C_4 (k_x L_{33} + k_x L_{22} + c_x L'_{33} + c_x L'_{22})$ $C_2 = (k_x H_{21} + c_x H'_{21}) - C_3^{-1} (k_{\theta} N_{21} + c_{\theta} N'_{21}) \times (k_x L_{33} + k_x L_{22} + c_x L'_{33} + c_x L'_{22})$ $C_3 = k_{\theta} P_{33} + k_{\theta} P_{22} + c_{\theta} P'_{33} + c_{\theta} P'_{22} + 1;$ $C_4 = k_{\theta} N_{33} + k_{\theta} N_{22} + c_{\theta} N'_{33} + c_{\theta} N'_{22}$

TABLE 2 FRF measured for different cutter with impact and string test

<i>Case</i>	<i>Cutter Length</i> (mm)	<i>Cutter Diameter</i> (mm)	<i>Impact Test</i>		<i>String Test</i>	
			$\min(\text{Re}[G_{11}])$ ($\times 10^{-6}$ m/N)	ω_n^1 (Hz)	$\min(\text{Re}[G_{11}])$ ($\times 10^{-6}$ m/N)	ω_n^1 (Hz)
1	95	12.70	21.4	392	23.2	397
2	68	12.70	17.6	535	16.4	512
3	42	12.70	7.7	1334	8.2	1297
4	42	9.525	11.7	935	12.6	938
5	42	6.350	19.0	512	18.9	499

TABLE 3 Connection coefficients of RCSA obtained with impact and string test

<i>Test</i>	k_x (N/m)	k_θ (N-m/rad)	c_x (N-s/m)	c_θ (N-m-s/rad)
<i>Impact</i>	3.80×10^6	2.44×10^5	1.62×10^2	1.12×10^{-1}
<i>String</i>	3.29×10^6	1.95×10^5	2.33×10^3	2.10×10^1

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a) schematic diagram of the experimental set-up

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FIGURE 7 Hexagonal attachment for measuring machine tool receptance at, (a) SHC; (b) cutter tip

(a)

(b)

FIGURE 8 Effect of tool on machine tool receptance, (a) magnitude of receptance at SHC and cutter tip; (b) real part of receptance at SHC and cutter tip

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