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ANALYZED

ABSORPTION OF SOUND BY A STRIP OF
ABSORPTIVE MATERIAL IN A DIFFUSE SOUND FIELD

BY

T. D. NORTHWOOD, M. T. GRISARU AND M. A. MEDCOF

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ERRATA: ABSORPTION OF SOUND BY A STR
OF ABSORPTIVE MATERIAL IN A DIFFUSE
SOUND FIELD

J. Acoust. Soc. Am. 31, 595 (1959)

T. D. Northwood, M. T. Grisaru, and M. A.
Division of Building Research,
National Research Council,
Ottawa -- Canada.

- (1) On page 597, the upper limits of integr
following Eq. (100) should be $k a \sin \theta$
- (2) In the paragraph containing Eq. (102),
value of A is $(1 - \sin^2 \theta)^{-\frac{1}{2}} = 1/\cos \theta$
- (3) On page 597, above Eq. (100), the eff
ratio should be $\eta' = \eta k/K = \eta/\sin \theta$
- (4) On page 597, Eq. (100) the $\sin \theta$ in
should be $\sin^2 \theta$.
- (5) In Fig. 5, the two sets of experimen
Sample 3 are interchanged, i.e.
• denotes Sample 1
• denotes Sample 2
- (6) In Eq. (38) the last factor, $\text{Re}(\eta)$
denominator, i.e. $\sigma_s/\sigma_a = |\eta|^2$

ANALYZED

Absorption of Sound by a Strip of Absorptive Material in a Diffuse Sound Field*

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(Received February 5, 1959)

The random incidence sound absorption coefficient is calculated for a narrow strip of absorbing material set in an otherwise reflecting plane. The assumptions made are that the material is of the locally reacting type, with a real normal admittance. The effect of an imaginary component of admittance is discussed qualitatively.

The results show the increase in absorption coefficient that occurs at small widths due to diffraction. Consideration is also given to the absorption of rectangular patches having both dimensions in the diffraction range. The standard reverberation room measurement of sound absorption is examined.

INTRODUCTION

THE performance of a sound absorbing material is generally determined by placing a sample in a reverberation room and observing its effect on the reverberation time in the room. The sound absorption coefficients for the material are then calculated from the two sets of reverberation times using the Sabine or Norris-Eyring reverberation formula. The coefficients thus obtained may be used with reasonable success in the many acoustical problems for which the reverberation theory is valid. There is, however, one notable weakness in the procedure. The sample that can conveniently be tested in existing reverberation rooms is a relatively small patch the dimensions of which are a few wavelengths or less, depending on frequency. In this region diffraction effects may be expected to occur, resulting in measured absorption coefficients that depend on the dimensions of the sample. It is desirable to know the extent of this effect, and to be able to correct for it in typical applications.

Most of the laboratories that operate reverberation rooms have made experimental studies of the effect of sample size. Of the published material¹⁻³ the most

comprehensive is Chrisler's well-known paper. There are difficulties, however, in studying an adequate range of sample sizes within the confines of established reverberation room techniques. In particular, it is difficult to approach the limiting value corresponding to an infinite area. To achieve the diffuse field which is basic to the reverberation theory, the total absorption in the room must not become too high; and to avoid effects peculiar to the room boundaries the sample must not approach closer than about half a wavelength to an edge or corner of the room.⁴ For very small samples, the only difficulty is in producing a measurable change in the absorption in the room. One may avoid this difficulty by using several small samples, but there will probably be an interaction unless they are spaced several wavelengths apart.

One of the earliest theoretical studies of the problem was by Pellam,⁵ who developed an expression for the absorption of an infinitely long narrow strip for plane waves incident in a direction perpendicular to the axis of the strip. Levitas and Lax⁶ also investigated this problem, and set up a variational method applicable to more complicated cases, one of which will be dealt with in this paper. Both of these studies apply to a material of the "locally reacting" type the behavior of which can be specified by its specific normal impedance or admittance.

* This paper is a contribution of the Division of Building Research, National Research Council, Canada, and is published with the approval of the Director of the Division.

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¹ V. L. Chrisler, J. Research Natl. Bur. Standards 13, 169 (1934).

² Paul Sabine, J. Acoust. Soc. Am. 6, 239 (1935).

³ Norris, Nixon, and Parkinson, J. Acoust. Soc. Am. 9, 234 (1938).

⁴ R. V. Waterhouse, J. Acoust. Soc. Am. 29, 544-547 (1956).

⁵ J. R. Pellam, J. Acoust. Soc. Am. 11, 396 (1940).

⁶ A. Levitas and M. Lax, J. Acoust. Soc. Am. 23, 316-322 (1951).

FIG. 1. Geometrical formulation of the problem. The strip lies in the XY plane, its center line coinciding with the Y axis. OP is the propagation vector k for a typical incident plane wave, with components k_x , k_y , and k_z . OA is the quantity referred to in the text as $\kappa = (k^2 - k_y^2)^{1/2} = k \sin \phi$.

Cook⁷ has considered both the strip and the circular patch, and has worked out the latter for both normal and random incidence. But this was achieved at the expense of assuming a piston absorber, the whole surface of which must act in phase, and except for normally incident sound the absorption coefficient must approach zero for large dimensions. Hence the results for random incidence do not throw much light on the practical case. More recently Cook⁸ has been able to generalize the method by replacing the piston absorber with one in which the surface motion may be specified by an appropriate Fourier expansion. The components may be selected to fit the boundary condition of a specified normal admittance. This work, when completed, will represent a complete solution to the problem, exact in form although it will involve the evaluation of series solutions.

The present work is a direct extension of the method of Levitas and Lax to determine the random incidence absorption of a strip. The procedure will be developed very briefly with frequent reference to their paper. They consider an absorbing strip of infinite length set in an otherwise perfectly reflecting plane, the XY plane (Fig. 1). The center line of the strip coincides with the Y axis, and its width is a . Its absorptive properties are specified by a normal admittance ratio η (eventually assumed to be real). The first step in the solution is to use Green's-function techniques to express the pressure at any point as an integral involving the pressure on the surface of the strip. This leads to an integral expression for the pressure on the strip itself. No attempt is made to solve this, but it is used in a variational procedure leading to expressions for the sum and the ratio of scattering and absorption cross sections. These are combined to yield explicit expressions for scattering and absorption cross sections, in terms of wavelength, width of strip, and its normal

admittance ratio. The solution is developed for propagation in the XZ plane, but it is shown that incidence out of the XZ plane may be dealt with by considering only the component normal to the Y axis (since the absorption of the material is assumed to depend only on the normal impedance).

THEORY FOR OBLIQUE INCIDENCE

Levitas and Lax work out the complete solution for normal incidence only. The more general case is readily evaluated by retaining the angular variable in their Eq. (36) and thereafter. Incidence in the XZ plane is first considered. Equations (38) and (49) retain the same form except that the integrals A and B are modified by a $\cos(x \sin \theta)$ factor relating to the normal impedance assumption [see Eq. (100) below].

Thus

$$\sigma_s/\sigma_a = |\eta|^2 A \operatorname{Re}(\eta) \quad (38)$$

and

$$\sigma_t = \sigma_s + \sigma_a = 4a \operatorname{Re} \left(\frac{\eta}{1 + \eta(A + iB)} \right), \quad (49)$$

where σ_s , σ_a , and σ_t are, respectively, the scattering, absorption, and total cross sections (per unit length).

FIG. 2. Absorption coefficient of infinite strip of width a and real admittance ratio δ .

⁷ R. K. Cook, J. Acoust. Soc. Am. **29**, 324-329 (1957).

⁸ R. K. Cook, J. Acoust. Soc. Am. **30**, 697 (A) (1958).

§ Equations numbered below (100) are from Levitas and Lax (reference 6).

Combining these, putting $\eta = \delta + i\xi$,

$$\sigma_a(\theta) = \frac{4a\delta}{1 + (\delta^2 + \xi^2)(A^2 + B^2) + 2(\delta A - \xi B)}.$$

Following Levitas and Lax, this expression is modified to take account of incidence out of the XZ plane by replacing η with an effective admittance ratio

$$\eta' = \eta k / \kappa = \eta \sin \phi.$$

This amounts to considering only the component of the incident wave parallel to the XZ plane for calculating both incident and absorbed energy. To obtain the absorption cross section, defined as the energy absorbed from an incident wave of unit intensity, the expression must be multiplied by $\sin \phi$ to restore the variation of incident energy with angle ϕ . The limits of the integrals A and B are also affected since $d = \frac{1}{2}ka = \frac{1}{2}ka \sin \phi$. With these changes, the absorption cross section is expressed as a function of θ and ϕ

$$\sigma_a(\theta, \phi) = \frac{4a\delta \sin \phi}{(\delta^2 + \xi^2)(A^2 + B^2) + 2 \sin \phi (\delta A - \xi B) + \sin^2 \phi}, \quad (100)$$

where

$$A = \int_0^{\pi/2} \cos(x \sin \theta) J_0(x) dx - \frac{1}{ka \sin \phi} \int_0^{\pi/2} x \cos(x \sin \theta) J_0(x) dx$$

FIG. 3. Infinite-area absorption coefficient as a function of δ and ξ (real and imaginary parts of complex admittance ratio).

and

$$B = \int_0^{\pi/2} \cos(x \sin \theta) N_0(x) dx - \frac{1}{ka \sin \phi} \int_0^{\pi/2} x \cos(x \sin \theta) N_0(x) dx.$$

This is the energy absorbed by a unit length of strip for an incident plane wave of unit intensity.

Random Incidence Absorption Coefficient

Consider now an incident sound field consisting of a uniform distribution of such incident waves (of unit intensity) for all angles in the hemisphere. The incident energy density on the strip for one such wave will be $\cos \theta \sin \phi$. The energy per unit area absorbed from such a wave will be $\sigma_a(\theta, \phi)/a$. The random incidence absorption coefficient is obtained by integrating each of these quantities over the hemisphere and forming the quotient (total energy absorbed per unit area)/(total incident energy density). The denominator is just $\pi/4$, so that

$$\bar{\alpha} = \frac{16\delta}{\pi} \int_0^{\pi/2} \int_0^{\pi/2} \frac{\sin^2 \phi \cdot \sin \phi d\phi d\theta}{(\delta^2 + \xi^2)(A^2 + B^2) + 2 \sin \phi (\delta A - \xi B) + \sin^2 \phi}. \quad (101)$$

When a approaches zero, A and B also approach zero and Eq. (101) becomes $\bar{\alpha}_0 = 8\delta$. This is the value usually obtained for a strip or patch of absorbing material small enough that it does not significantly alter the sound field at the otherwise reflecting surface.

When the strip width becomes very large $B \rightarrow 0$ and $A \rightarrow (1 - \sin \theta)^{-1} = 1/\cos \theta$. Equation (101) then reduces to the more symmetric form

$$\bar{\alpha}_\infty = \frac{16\delta}{\pi} \int_0^{\pi/2} \int_0^{\pi/2} \frac{\cos^2 \theta \sin^2 \phi \cdot \sin \phi d\phi d\theta}{(\delta^2 + \xi^2) + 2\delta \cos \theta \sin \phi + \cos^2 \theta \sin^2 \phi}. \quad (102)$$

Noting that the term $\cos \theta \sin \phi = \cos I$, where I is the angle of incidence, the integral is readily evaluated by rotating the coordinates so that the polar axis is normal to the surface. The result is

$$\bar{\alpha}_\infty = 8\delta \left\{ 1 - \delta \ln \left(\frac{1 + 2\delta + \delta^2 + \xi^2}{\delta^2 + \xi^2} \right) + \frac{\delta^2 - \xi^2}{\xi} \tan^{-1} \left(\frac{\xi}{\xi^2 + \delta^2 + \delta} \right) \right\}. \quad (103)$$

This agrees with Morse's expression⁹ for the absorption

⁹ P. M. Morse, *Vibration and Sound* (McGraw-Hill Book Company, Inc., New York, 1948), second edition, p. 388.

FIG. 4. Experimental measurements of $\bar{\alpha}$ for long strips and approximately square patches of same material. (Samples described in Table I. Curves are theoretical results.)

coefficient of an infinite area. For real admittance this reduces to

$$\bar{\alpha}_{\infty}' = 8\delta \left\{ 1 - 2\delta \ln \left(\frac{1+\delta}{\delta} \right) + \frac{\delta}{1+\delta} \right\}. \quad (104)$$

Equation (101) has been evaluated for a range of values of ka and for real admittance, with the results shown in Fig. 2. Values for infinite strip width, calculated from Eq. (104), are plotted on the right-hand ordinate.

It will be seen that the absorption coefficient increases continuously as the width decreases, without the periodic fluctuations of the normal-incidence case. This is an expected consequence of including obliquely incident waves in the calculation. Another result of the oblique incidence is the fact that the effect is significant even when the strip is several wavelengths wide.

Some information on the effect of an imaginary component of admittance may be obtained by considering the formulas for very narrow and very wide strips. For very narrow strips the absorption coefficient depends only on the real part. For infinite width the absorption is given by Eq. (103), from which the curves of Fig. 3 are plotted, and it will be seen that the coefficient is decreased by the presence of an imaginary component. Thus, for a complex admittance the curves will start as in Fig. 2, depending on the real part of

the admittance, and for large widths will droop lower than in Fig. 2 by an amount depending on the magnitude of the imaginary part. Hence the curves of Fig. 2, for real admittances, indicate the minimum variation of absorption with strip width that can occur.

Absorption of a Rectangular Patch

The results for an infinite strip presumably can be applied directly to finite strips when the length is greater than a few wavelengths. It is important to know the variation that occurs when *both* dimensions of a rectangular patch are small enough to produce diffraction effects. An empirical way of adapting the infinite strip results to this case is to replace the strip width by a parameter that depends equally on the two dimensions of the rectangle and which reverts to the strip formula when either dimension becomes very large. The simplest procedure is to replace a by $bc/(b+c)$, where b and c are the dimensions of the rectangle. This gives the correct limiting values and will not be greatly in error when the patch takes the form of a long but finite strip. The most serious deviation is likely to be for a square patch, which is given the same diffraction effect as an infinite strip of width equal to half the side of the square. Experimental studies with squares and long strips indicate that this empirical formula is a good approximation (Fig. 4).

FIG. 5. Experimental measurements of $\bar{\alpha}$ for rectangular patches of dimensions $b \times c$. (Samples described in Table I. Curves are theoretical results. Points in parentheses are calculated.)

EXPERIMENTAL EVIDENCE

Over the years considerable experimental information has been accumulated by the senior author on the variation of absorption with area. For three materials impedance tube measurements were also available, and since the materials were mounted in the same way for both measurements the results should be consistent. Experimental results for these materials are plotted in Figs. 4 and 5, with the theoretical curves of Fig. 2 for comparison.

In Fig. 4 three sets of results for long strips and nearly square patches of the same material are shown, demonstrating that the empirical formula for rectangular patches is reasonably independent of sample shape. Additional results are plotted in Fig. 5.

Table I gives impedance tube and other information for the three samples. Values of $\bar{\alpha}_0$ and $\bar{\alpha}_\infty$, corresponding to very small and very large areas, are given in the table and are also plotted in Figs. 4 and 5, where they may be compared with the experimental points. Comparison of $\bar{\alpha}_\infty$ and $\bar{\alpha}_\infty'$ (obtained by neglecting ξ)

TABLE I. Measurements and calculations for experimental materials.

Sample	Material	Freq. cps	δ	ξ	Tube data ^a			Room data Approx. δ
					$\bar{\alpha}_0$	$\bar{\alpha}_\infty$	$\bar{\alpha}_\infty'$	
1	Glass fiber 2 in. thick	500	0.31	-0.47	2.5	0.70	0.85	0.4
		1000	0.84	-0.39	6.7	0.88	0.94	0.8
2	Perforated wood fiber 1½ in. thick	500	0.37	-0.22	2.56	0.86	0.89	0.35
		1000	0.23	+0.13	1.84	0.75	0.77	0.17
3	Perforated wood fiber ¾ in. thick	500	0.09	-0.03	0.71	0.47	0.48	0.12
		1000	0.14	-0.03	1.12	0.60	0.60	0.18

^a Normal admittance ratio $\eta = \delta + i\xi$.

indicates the deviation from the real admittance curves that may be expected for large areas. The last column is obtained by comparing the experimental results with the theoretical curves and estimating the admittance ratios for the samples.

More experimental work is planned, using more carefully designed samples. In the meantime the agreement is good enough to suggest that for certain types of material, the random incidence absorption can be predicted from impedance tube information. The practical significance of this is limited by the fact that the absorption of many materials depends on large-scale effects such as mounting and backspace. One must also rule out materials for which the normal impedance

FIG. 6. Infinite-area absorption vs absorption of standard sample (9 ft by 8 ft).

assumption does not apply. Nevertheless the fact that there appears to be correspondence where there should be helps to close the gap that has existed between impedance tube measurements and reverberation room results.

AREA EFFECT AND REVERBERATION ROOM MEASUREMENTS

For sound absorption tests most reverberation rooms on this continent use a standard sample having the dimensions 9 ft by 8 ft. Applying the foregoing results to a patch of these dimensions gives the curves of Fig. 6, which relate standard-area coefficients and infinite-area coefficients. The effect is a substantial one, which should not be ignored. There are two possible corrective procedures. One can correct standard-area measurements to the infinite-area values, or one can continue to quote standard-area measurements in the expectation that the user will calculate the absorption of each given area. Possibly both values might be given, the infinite-area one for routine calculations and the standard-area value to be used as a starting point when more precise calculations are required.

R. K. Cook has suggested that the term "absorption coefficient," implying a material property that is independent of area, might be replaced by the "absorption cross section," which is simply the total absorption of a given patch of material. For routine applications involving large areas of treatment perhaps this would be an unduly complicated procedure. But for patches of less than a few wavelengths in dimension, or for special configurations such as space absorbers, the absorption cross section is a useful quantity.

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